Abstract:
The concept of sub-Riemannian geometry will be introduced and a possible way to generalize it to infinite-dimensional manifolds will be explained. Shortly, a sub-Riemannian manifold is a smooth manifold $M$ with a given sub-bundle $D$ of the tangent bundle, and with a metric defined on the sub-bundle $D$. We present the structure of geodesics, tangent to $D$, and compare them in the infinite- and the finite-dimensional cases. Infinite-dimensional Lie groups are of special interest. As examples, we consider the group of sense-preserving diffeomorphisms of the unit circle and the Virasoro-Bott group with their respective special sub-bundles, which are related to the space of normalized univalent functions. We show that any two points in these groups can be connected by a curve tangent to the chosen sub-bundle. We present formulas for geodesics for different choices of metrics. The geodesic equations are analogues to Camassa-Holm, Huter-Saxton, KdV, and other known non-linear PDE.
Evolution of smooth shapes and integrable systems

Abstract:
We consider a homotopic evolution in the space of smooth shapes starting from the unit circle. Based on the Loewner-Kufarev equation we give a Hamiltonian formulation of this evolution and provide conservation laws. We study an embedding of the Loewner-Kufarev trajectories into the Segal-Wilson Grassmannian, construct the tau-function, the Baker-Akhiezer function, and finally, give a class of solutions to the KP equation.
Graded contact manifolds and principal Courant algebroids

Abstract:
Contact structures will be interpreted as symplectic principal $GL(1,R)$-bundles. Gradings compatible with the $GL(1,R)$-action lead to the concept of a graded contact manifold, in particular, a linear contact structure. Linear contact structures will be proven to be exactly the canonical contact structures on first jets of line bundles. They give rise to linear Kirillov (or Jacobi) brackets and the concept of a principal Lie algebroid, a contact analog of a Lie algebroid. One can view Kirillov or Jacobi brackets as homological Hamiltonians on linear contact manifolds. Contact manifolds of degree 2, called principal Courant algebroids, will be presented as contact analogs of Courant algebroids.