MATHEMATICS SEMINAR
of the
UNIVERSITY OF LUXEMBOURG
in cooperation with the
LUXEMBOURG MATHEMATICAL SOCIETY

October 2006

24 October 2006 at 5 pm **Room 3.04 bs**

Oleg Sheinman
Steklov Institute Moscow

Infinite-dimensional Lie algebras related to holomorphic bundles on Riemann surfaces

We will define the algebras of Lax operators related to an arbitrary Riemann surface and algebra $gl(n)$ (construction due to I.Krichever), and present its generalization on $so(n)$ and $sp(2n)$ (joint result of I.Krichever and the author). We will consider central extensions of those algebras and comment on their relation to Atiyah algebras introduced by Beilinson and Schechtman. The talk continues the one I have given in Luxembourg last year. I will comment on advances in the subject and problems to be resolved.

31 October 2006 at 5 pm **Room 3.04 bs**

Martin Olbrich
University of Luxembourg

Dynamical zeta functions and representation theory

We discuss Ruelle and Selberg zeta functions, which are associated with the geodesic flow of negatively curved locally symmetric spaces. Initially, they are defined on a certain complex half plane. The main issue will be the understanding of the singularities of the meromorphic continuation of these functions. We present results and conjectures providing such an understanding in terms of representations of the group of isometries of the corresponding globally symmetric space.

Mathematics Seminar
of the
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Luxembourg Mathematical Society

November 2006

7 November 2006 at 5 pm **Room 3.04 bs**

Anton Thalmaier
University of Luxembourg

Brownian motion and Riemann zeta function

Abstract

The theory of Brownian motion contains many remarkable identities. Some of them were considered as curiosities, though eventually they received a complete explanation inside a broader theory; for others a proper explanation remains to be found. We discuss some of such formulas related to the Jacobi theta and Riemann zeta functions, Bessel processes and Brownian excursions. Possible connections to Statistical Mechanics (ferromagnetic spin systems, Lee-Yang-Newman theorem) are sketched.

14 November 2006 at 5 pm **Room 3.04 bs**

Eli Hawkins
Radboud University, Nijmegen

A Groupoid Approach to Quantization

Abstract

Symplectic groupoids were introduced almost 20 years ago as a tool for the geometric quantization of Poisson manifolds. Unfortunately, in this time there has been relatively

little progress on this approach to quantization. The problem is that most polarizations do not respect the groupoid structure.

I introduce the concept of a groupoid polarization, and in particular a symplectic groupoid polarization. Using this, I sketch a geometric quantization procedure for Poisson manifolds. This appears to unify all examples of geometrically constructed quantizations of Poisson manifolds.

21 November 2006 at 5 pm **Room 3.04 bs**

Marc Arnaudon
University of Poitiers

Coupling semimartingales in manifolds; some recent applications

Abstract

We report on recent work where parallel coupling of Brownian motions with drift is used to prove geometric inequalities on manifolds. We compare the densities of the joint law of hitting time and hitting position of the boundary of a relatively compact domain, by two Brownian motions started at different points inside the domain. We use it to obtain Harnack inequalities and estimates of the heat kernel in manifolds with curvature unbounded below.

22 November 2006 at 5 pm **Room 3.04 bs**

Claude Roger
University of Lyon

The Schrödinger-Virasoro Lie Group and Algebra: Representation Theory and Cohomological Study

Abstract

The talk will focus on the Schrödinger-Virasoro Lie algebra, an infinite-dimensional Lie algebra, introduced in the frame of non-equilibrium statistical physics, and containing as subalgebras both, the Lie algebra of invariance of the free Schrödinger equation and the central charge-free Virasoro algebra. Realizations as Lie symmetries of field equations, coadjoint representation, coinduced representations in connection with Cartan's prolongation method, as well as cohomological investigations, in particular a classification of deformations and central extensions, will be addressed.

28 November 2006 at 5 pm **Room 3.04 bs**

Charles Boubel
University of Strasbourg

Lorentzian flows on compact 3-manifolds

Abstract

I will give a classification of the 1-dimensional foliations —here shortly called flows— of compact 3-manifolds, which are transversally Lorentzian and satisfy a transversal completeness condition. In the main part of the talk we will then present a new example of a Lorentzian flow which is not transversally complete.

Let me recall that a pseudo-Riemannian metric transverse to a foliation F on a manifold M is a field of non degenerate symmetric bilinear forms on the normal bundle TM/TF of F, which is invariant by the flow of every vector field tangent to F. If we take for F the trivial foliation of M by points, we get the definition of a pseudo-Riemannian metric on M. The existence of the latter depends on purely topological conditions on M and always holds in the Riemannian case. The existence of a transversal metric to a non trivial foliation F, on the other hand, yields strong dynamical conditions on F.

This is a joint work with P. Mounoud and C. Tarquini.

Mathematics Seminar
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December 2006

5 December 2006 at 2 pm **Room 3.04 bs**

Mathematics Colloquium
(lecture in the frame of the Seminar for doctorands)
Roland Friedrich
Max-Planck-Institut Bonn

Stochastic Loewner Evolutions and some of its Ramifications

Abstract

In this talk we shall give an overview of various aspects of Stochastic Loewner Evolutions (SLE). SLE developed spectacularly in the last couple of years, starting with an ingenious idea of how to describe possible scaling limits of various discrete 2D models. So, in particular we shall talk about the underlying and unifying framework that permitted to generalize it to non trivial topologies as e.g. arbitrary domains or surfaces, or to singular geometries like polygons. Further we shall also briefly mention its connections with representation theory of the Virasoro algebra and Conformal Field Theory. The talk is of interest to a broader mathematical or physical audience.

5 December 2006 at 5 pm **Room 3.04 bs**

Johannes Huebschmann
Université des Sciences et Technologies de Lille

Singular Poisson-Kähler geometry of stratified Kähler spaces

Abstract

A *stratified Kähler space* is a stratified symplectic space together with a complex analytic structure which is compatible with the stratified symplectic structure; in particular

each stratum is a Kähler manifold in an obvious fashion. The notion of stratified Kähler space establishes an intimate relationship between nilpotent orbits, singular reduction, invariant theory, reductive dual pairs, Jordan triple systems, symmetric domains, and prehomogeneous spaces. The purpose of the talk is to illustrate the significance of stratified Kähler spaces.

Examples of stratified Kähler spaces abound. The closure of a holomorphic nilpotent orbit carries a normal Kähler structure. Symplectic reduction carries a Kähler manifold to a normal stratified Kähler space in such a way that the sheaf of germs of polarized functions coincides with the ordinary sheaf of germs of holomorphic functions. Projectivization of holomorphic nilpotent orbits yields exotic stratified Kähler structures on complex projective spaces and on certain complex projective varieties including complex projective quadrics. Other examples come from certain moduli spaces of holomorphic vector bundles on a Riemann surface and variants thereof; in physics language, these are spaces of conformal blocks. Still other physical examples are reduced spaces arising from angular momentum.

In the world of singular Poisson-Kähler geometry, reduction after quantization coincides with quantization after reduction: For a stratified symplectic space, the concept of stratified polarization, which is defined in terms of an appropriate Lie-Rinehart algebra, encapsulates polarizations on the strata and, moreover, the behaviour of the polarizations across the strata. Exploiting the notion of stratified Kähler space, one can prove that, given a Kähler manifold, reduction after quantization coincides with quantization after reduction in the sense that not only the reduced and unreduced quantum phase spaces correspond but the invariant unreduced and reduced quantum observables as well.

12 December 2006 at 5 pm **Room 3.04 bs**

Mathematics Colloquium
(lecture in the frame of the Mathematics Seminar)

Karl-Theodor Sturm
University of Bonn

Optimal Transportation, Ricci Curvature and Diffusions on the L2-Wasserstein Space

Abstract

We introduce and analyze generalized Ricci curvature bounds for metric measure spaces (M, d, m), based on convexity properties of the relative entropy $\mathrm{Ent}(\cdot|m)$. For Riemannian manifolds, $\mathrm{Curv}(M, d, m) \geq K$ if and only if $\mathrm{Ric}_M \geq K$ on M; for the Wiener space, $\mathrm{Curv}(M, d, m) = 1$.

One of the main results is that these lower curvature bounds are stable under (e.g. measured Gromov-Hausdorff) convergence. This solves one of the basic problems in this field, open for many years.

Furthermore, we introduce a (more restrictive) curvature-dimension condition $\mathrm{CD}(K, N)$ which implies sharp versions of the Brunn-Minkowski inequality, of the Bishop-Gromov volume comparison theorem and of the Bonnet-Myers theorem. Moreover, it allows to construct a canonical Dirichlet form with Gaussian bounds for the corresponding heat kernel.

Finally, we indicate how to construct a canonical reversible process on the L^2-Wasserstein space of probability measures $\mathcal{P}(\mathbb{R})$, regarded as an infinite dimensional Riemannian manifold. This process has an invariant measure $\mathbb{P}^\beta$ which may be characterized as the 'uniform distribution' on $\mathcal{P}(\mathbb{R})$ with weight function $\frac{1}{Z}\exp(-\beta \cdot \mathrm{Ent}(\cdot|m))$ where m denotes a given finite measure on $\mathbb{R}$. One of the key results is the quasi-invariance of this measure $\mathbb{P}^\beta$ under push forwards $\mu \mapsto h_*\mu$ by means of smooth diffeomorphisms h of $\mathbb{R}$.

13 (!) December 2006 at 11.15 am **Room 3.04 bs**

Mathematics Colloquium
(lecture in the frame of the Probability Seminar)

Max von Renesse
TU Berlin

Entropic Measure and Wasserstein Diffusion of Probability Measures on the Unit Interval

Abstract

This is a report on recent results obtained jointly with K-T Sturm (Bonn). We construct a diffusion process on the space of probability measures on the unit interval by Dirichlet form methods. For this we construct a Gibbs type invariant measure with the Boltzmann entropy function as Hamiltonian. We show an integration by parts formula for this measure and demonstrate, that choosing the appropriate notion of a gradient, the intrinsic metric of the process is the quadratic Wasserstein distance.

18 (!) December 2006 at 5 pm **Room 3.04 bs**

Robert Wolak
Jagiellonian University of Krakow

Geometry of singular spaces and foliations

Abstract

We will show how singular stratified spaces arise naturally as leaf closure spaces of singular Riemannian foliations. Then we discuss the relation between foliated structures and geometric structures on the leaf closure space giving conditions which assure that given structures project onto the leaf closure space. The foliated space can be considered as a desingularization of its leaf space. This class includes the orbit spaces of compact group actions and orbifolds in particular. We shall discuss Riemannian, Kähler, symplectic, Poisson and Finsler structures as well as linear connections and (transversally) harmonic maps.