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Confronting the Representative Consumer with Household-Size Heterogeneity

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Confronting the Representative Consumer with Household-Size Heterogeneity*

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Abstract

Most simulated micro-founded macro models use solely consumer-demand aggregates in order to estimate deep economy-wide preference parameters, which are useful for policy evaluation. The underlying demand-aggregation properties that this approach requires, should be easy to empirically disprove: since household-consumption choices differ for households with more members, aggregation can be rejected if appropriate data violate an affine equation regarding how much individuals benefit from within-household sharing of goods. We develop a survey method that tests the validity of this equation, without utility-estimation restrictions via models. Surprisingly, in six countries, this equation is not rejected, lending support to using consumer-demand aggregates.

Keywords: Linear Aggregation, Dynamic Representative Consumer, Household-Size Economies, Equivalent Incomes, Survey Method

JEL classification: C42, E21, D12, E01, D11, D91, D31, I32

1. Introduction

Most simulated micro-founded macro models are combined with aggregate-consumption time series in order to estimate deep economy-wide preference parameters.¹ These models make an implicit aggregation assumption: that there is a direct channel enabling incentives behind consumption choices to be transmitted from the micro level to the macro level and to be reflected in aggregate consumption fluctuations. The strong form of exact linear aggregation which implies the existence of a representative consumer (RC) has also been used as a working hypothesis in existing macroeconomic studies with heterogeneous agents in order to analyze observed consumer choices in a more tractable way.² Since most consumption data are collected at the household level, and since incentives behind household-consumption choices differ for households with more members, it is reasonable to think that aggregation can fail just because of household-size heterogeneity. In this paper, (i) we theoretically identify the comprehensive class of heterogeneous utility functions which are time-separable and time-varying, and which allow for linear aggregation in dynamic environments where households

¹ Such economy-wide preference parameter estimates are further used for forecasting and for policy evaluation.

² A study clarifying the tractability of aggregation in the analysis of heterogeneous-agent models is Caselli and Ventura (2000), which builds on previous findings by Chatterjee (1994). This tractability is also the starting point of the crucial review paper about doing quantitative macroeconomics with heterogeneous households by Heathcote, Storesletten and Violante (2009), e.g. in their subsection “Heterogeneity with complete markets”. There are cases of aggregation in which preference heterogeneity among utility maximizers is combined with specific distributional properties of consumer characteristics that enter the budget constraint (wealth, income, skill distributions, etc. – for a review, see Blundell and Stoker, 2005). On the contrary, exact aggregation does not depend on features of wealth/income distributions at all. So, to instill exact aggregation properties to a heterogeneous-agent model means to use particular functional forms for the utility function(s) of households, which guarantee the existence of an RC: a fictitious agent who is always endowed with the aggregate resources of a heterogeneous-agent economy and whose choices always coincide with economy-wide aggregated choices *under any price regime and under any underlying distribution of resources at the individual or household level*. So, if \mathcal{I} is a set of households with different utility functions $\{U^i\}_{i \in \mathcal{I}}$, then aggregation means being able to use $\{U^i\}_{i \in \mathcal{I}}$ as ingredients for constructing a utility function of RC, i.e. $U^{RC}|_{\{U^i\}_{i \in \mathcal{I}}}$, which is maximized subject to the aggregated economy-wide resource constraints and always replicates aggregated choices by the community having $\{U^i\}_{i \in \mathcal{I}}$, when solutions are interior. Section 2 of this paper is devoted to understanding the necessary and sufficient functional properties of $\{U^i\}_{i \in \mathcal{I}}$ that enable the construction of $U^{RC}|_{\{U^i\}_{i \in \mathcal{I}}}$.

have savings options and dynasties exhibit changing family size over time, (ii) we devise a test which can empirically identify whether utility functions driving actual choices belong to the identified aggregation class, and which can be performed without utility-estimation restrictions via models, (iii) we design a survey method that generates appropriate data in order to perform the aggregation test in several countries, and (iv) we test the validity of the survey method itself.

In our analysis dynasties that accumulate wealth exhibit changing family size over time.³ We do not accommodate endogenous family formation, but our results should be useful for that extension.⁴ Here we only allow a household's momentary objective function to change over time. So, a novelty is that we provide necessary and sufficient conditions for exact linear aggregation, for any time-varying momentary utility functions.⁵ The comprehensive class of household objective functions which are consistent with exact linear aggregation is a generalized class of the Identical-Shape Hyperbolic-Absolute-Risk-Aversion (ISHARA) utility function, and of the exponential form in which particular parameters (subsistence levels or bliss points) are allowed to vary over time and across family types.⁶

³ A paper documenting the dramatic family-size changes in the US since the 19th century is Salcedo, Schoellman and Tertilt (2012, cf. Figure 1, p. 138).

⁴ There is a substantial body of macroeconomics literature that introduces family-formation decisions in dynamic frameworks. See, for example, Aiyagari et al. (2000), Greenwood and Seshadri (2002), Greenwood et al. (2005), Fernández et al. (2005), Fernández-Villaverde et al. (2011), Knowles (2013), among others.

⁵ To our knowledge, the only linear-aggregation study which allows for time-variant momentary utility functions is Caselli and Ventura (2000). Yet, Caselli and Ventura (2000) provide one class of utility functions which is sufficient for aggregation, while here we also provide necessary aggregation conditions which characterize all types of utility functions which lead to exact linear aggregation.

⁶ A set of utility functions, $\{u^i\}_{i \in \mathcal{I}}$, in a community belongs to the same ISHARA class if absolute risk aversion of a utility function $u^i(c)$, is a harmonic function $[\alpha c + \beta^i(t)]^{-1}$, i.e., the coefficient α multiplying consumption is the same for all $i \in \mathcal{I}$, and constant over time, while $\beta^i(t)$ differs across households and over time. Another recent study that provides necessary and sufficient conditions for aggregation in a dynamic environment is Mazzocco (2007, cf. Proposition 1). A key difference from our analysis here is that Mazzocco (2007) does not allow for demographic changes over time, so his momentary-utility functions are time-invariant. Given also that Mazzocco (2007) conducts his analysis in discrete time, there are similarities with Pollak's (1971) proofs, who finds necessary and sufficient aggregation conditions for static environments with multiple goods when individual utilities are additively-separable across different goods. On the contrary, our analysis accommodates that, whenever a family anticipates a change in its size, it will incorporate the

This aggregation result leads to a sharp testable implication. This implication involves permanent equivalent incomes (EIs): permanent household incomes that equate the level of material comfort of individuals living in households of different size. If utilities belong to the time-varying ISHARA and exponential family class that we identify as necessary and sufficient for aggregation, then each permanent EI is an affine transformation of any other household type’s permanent EI. The interpretation of this implication is that, once expenditures for subsistence needs are subtracted from permanent income, then the remainder permanent incomes must entail the same benefits from within-household sharing of goods for the rich and for the poor. Apparently, this result is a stringent condition.

In our empirical analysis we test this stringent condition. The testing relies on representative survey data from Germany (2,024 respondents), and on several small-scale surveys (conducted in countries as dissimilar as France, Cyprus, China, India, and Botswana). These surveys, introduced by Koulovatianos et al. (2005, 2009), provide peoples’ assessments of permanent EIs for several categories of rich, middle-class, and poor households.⁷ The data permit a total of 49 tests, and these do not reject the stringent aggregation condition regarding how much people benefit from within-household sharing. This finding does not prove the existence of a representative consumer. Yet it suggests that the sharing of goods within multimember households may not undermine the existence of a representative consumer.

Our theory and our empirical tests apply to both collective-bargaining household analy-

impact of this demographic change into its intertemporal Euler equation that governs its savings. Examples of studies that examine *only sufficient conditions* for aggregation in dynamic environments of consumer choice are Chatterjee (1994), Atkeson and Ogaki (1996), Caselli and Ventura (2000), and Maliar and Maliar (2001, 2003). Finally, there is a strand of literature that numerically examines whether “approximate aggregation” holds (see also Heathcote, Storesletten and Violante (2009) for further discussion). For example, Krusell and Smith (1998) and Carroll (2000) examine cases of heterogeneity in rates of time preference in incomplete-markets environments with Carroll (2000) focusing on conditions under which “approximate aggregation” fails.

⁷ Specifically, our survey ask questions of the form: “What is the net monthly household income that can make a household with two adults and two children attain the same level of material comfort as that of a one-member household with a net monthly income of \$3,000?”.

ses of the Mazzocco (2007) type, and to unitary-household analyses. The household-level utility functions we analyze can be seen as welfare functions among household members, irrespective of whether there is within-household bargaining or not. In addition, our theoretical work can offer insights on why a large body of literature using cross-sectional data, performing specification tests for the shape of good-specific Engel curves, is also unlikely to have offered final comprehensive evidence against aggregation. Building on household consumer/expenditure surveys, a number of studies suggest that Engel curves are not linear, at least for some goods.⁸ Linearity, however, is a necessary requirement for the existence of a representative consumer both in a static and in a dynamic framework. Yet, our aggregation results in a dynamic context suggest that the rejection of the Gorman requirement of linear and parallel Engel curve in empirical analyses may be due to missing variables. If households are forward looking, expectations of future events, e.g. changes in household income or composition, impact today’s consumer choices. If these expectations are not adequately modeled, the empirical tests have limited power.⁹ Our survey-based test focuses on a stringent linearity condition that pertains to economies from within-household sharing across the rich and the poor, and not on Engel-curve linearity tests. Since we do not test for Engel-curve linearity our results do not contradict but complement the findings of Cherchye et al. (2011). In future work, richer databases may provide a basis for more comprehensive aggregation tests.

Apart from being encouraging for macroeconomists who use aggregative models, our results are important for welfare and policymaking analysis. Most policies are likely to change both resource allocations and price patterns. Exact linear aggregation does not

⁸ See Blundell and Stoker (2005) for an extensive empirical literature review.

⁹ In Section 2 we demonstrate that consumption decision rules in a dynamic framework are extended Engel curves including other variables in addition to current income and current demographic status. Such additional variables are wealth, future-income anticipations, and anticipations of future demographic changes.

require any assumptions regarding the distribution of resources and does not impose any price-pattern restrictions. As Gorman (1953) proved, linear aggregation coincides with the requirement of non-intersecting social indifference curves. So, every time one changes policies in markets which have properties far away from exact linear aggregation, welfare and policy evaluation problems emerge due to potentially intersecting social indifference curves.¹⁰

2. Theoretical Results on Multidimensional Heterogeneity and the Existence of a Representative Consumer

In this section we fully characterize the class of utility functions of heterogeneous households that leads to the existence of a representative consumer: a fictitious consumer whose preferences represent an entire community-preference profile (the set of utility functions of all household types), and whose choices always coincide with actual aggregated choices under any price regime. Then, we show that the requirement that a representative consumer exists in the presence of household-size heterogeneity implies that equivalent incomes are necessarily linked through an affine transformation. Our goal is to accommodate the additional dimension of household-size heterogeneity. So, the question we make is: how much heterogeneity in household objective functions can the representative consumer survive?¹¹

¹⁰Specifically, Kirman (1992, p. 123-125) explains the (high) possibility of welfare misrepresentation through a picture, and Gorman (1955) provides an intuitive outline of the problems arising in the absence of aggregation. One of the most complete treatments of this welfare misrepresentation is Jerison (1994).

¹¹There has been vivid theoretical work on the topic of aggregation, and we can mention only a few key advances here. For a set of heterogeneous households that live for one period and decide once and for all about the consumption of different consumer goods, Gorman (1953) has shown that the indifference curves of a representative consumer are non-intersecting if, and only if, Engel curves for all traded commodities are always linear and parallel across all households for any given price regime. In a later study Gorman (1961) has shown that, for Engel curves to be linear and parallel, utility functions must meet a particular functional property; this property has led Pollak (1971) to a complete characterization of the set of utility functions of households that allow the existence of a representative consumer, under the assumption that all utility functions are additively separable with respect to each different good. Concerning households that act for more than one period, in particular for households that are infinitely-lived dynasties, previous work has focused on households that consume a single composite consumer basket and accumulate financial wealth over time: Chatterjee (1994) and Caselli and Ventura (2000) have identified household utility functions that are sufficient for the existence of a representative consumer. Here we complete their work by showing the

The core theoretical results on aggregation are split into two categories. The first category deals with dynasties where momentary utility functions are constant over time (Theorem 1).¹² Since, however, dynasties may experience changing demographic composition, Theorem 2 extends the analysis to time-variant momentary utility functions. Theorem 2 identifies which parameters of the resulting ISHARA utility functions are allowed to vary over time and which are not, which is a novel result of this paper. Yet, assumptions behind Theorem 2 are tighter compared to these of Theorem 1, and a part of the proof of Theorem 2 relies upon the proof approach of Theorem 1. For this reason, instead of moving to the statement and proof of Theorem 2 directly, we present the two results in two sections below.

2.1 Common Choice-Independent Rates of Time Preference and Time-Invariant Momentary Utility Functions

Time is continuous and the time horizon is infinite, $t \in [0, \infty)$. Households are all infinitely-lived and comprise a constant set \mathcal{I} of different types, with generic element i . The set of household types can be countable, finite, or a continuum. It can also be that all households are of the same type, and in any case there is a “large” number of households, making each of them having negligible impact on the aggregate economy, i.e., all households are price-takers.

Assume a measure $\mu : \mathcal{I} \rightarrow [0, 1]$, which has a density, $d\mu$, with,

$$\inf \{d\mu(i) \mid i \in \mathcal{I}\} > 0 . \tag{1}$$

So, if \mathcal{I} is finite, $d\mu(i) > 0$ for all $i \in \mathcal{I}$, whereas if \mathcal{I} is a compact interval, $d\mu(i)$ is continuous on \mathcal{I} and bounded away from 0. Households of different types can differ with

set of utility functions that is also *necessary* for the existence of a representative consumer (see Theorems 1 and 2 below). Mazzocco (2007, Proposition 1) has also provided necessary and sufficient exact-linear-aggregation conditions for time-invariant momentary utility functions in a dynamic economy, while here we extend analysis to time-variant momentary utility functions.

¹²Theorem 1 results from weak conditions and it is directly related to our empirical tests, since in our survey we ask respondents to evaluate hypothetical households with exogenously fixed demographic composition.

respect to their initial endowment of capital claims (assets) and also with respect to their labor productivity which is given by the exogenous function of time, $\theta^i : \mathbb{R}_+ \rightarrow \mathbb{R}_+$. Asset holdings for household $i \in \mathcal{I}$ at time 0 are denoted as a_0^i .¹³

There is a single composite consumable good. Household preferences of each $i \in \mathcal{I}$, are given by the general additively-separable utility function with a common across households rate of time preference captured by the positively-valued function $\rho : \mathbb{R}_+ \rightarrow \mathbb{R}_{++}$, where

$$U^i \left((c^i(t))_{t \geq 0} \right) = \int_0^\infty e^{-\int_0^t \rho(\tau) d\tau} u^i(c^i(t)) dt . \quad (2)$$

Assumption 1 For all $i \in \mathcal{I}$, $u^i : \mathbb{R}_+ \rightarrow \mathbb{R}$, is twice-continuously differentiable and such that $u_1^i(c) > 0$ and $u_{11}^i(c) < 0$ on some interval, $\mathbb{C}^i \subseteq \mathbb{R}_+$, with both $u_1^i(c) < \infty$ and $-\infty < u_{11}^i(c)$ for all $c \in \mathbb{C}^i \subseteq \mathbb{R}_+$, with $\underline{c}^i \equiv \inf(\mathbb{C}^i) < \sup(\mathbb{C}^i) \equiv \bar{c}^i$.

Assumption 1 secures that, for all $i \in \mathcal{I}$, there is a choice domain, $\mathbb{C}^i \subseteq \mathbb{R}_+$, which is an interval, and where standard desirable properties of momentary utility functions are present. Assumption 2 allows households to choose consumption paths such that, asymptotically, the consumption level is non-decreasing.

Assumption 2 $\int_0^\infty e^{-\int_0^t \rho(\tau) d\tau} dt < \infty$.

All households are endowed with the same amount of time at each instant, supplied for labor inelastically. The momentary time endowment is normalized to one, without leading to loss of generality: if a household is larger and more than one members work, given that

¹³With slight abuse of notation, we assume that a community preference profile given by a set of utility functions $(U^i)_{i \in \mathcal{I}}$, is fixed, while within a preference group $i \in \mathcal{I}$, there can be many individuals with heterogeneous initial endowments a_0^j , and labor-productivity functions, θ^j . Instead of distinguishing individuals across groups through a multi-dimensional measure (e.g. $\tilde{\mu}(i, j)$, where i denotes the utility group and j denotes a single household unit with (a_0^j, θ^j)), we resort to the reduced notation recommended above.

labor supply is inelastic, personal labor incomes within the household can be summed up and the household's total labor income can be used instead.

For any given price vector $(r(t), w(t))_{t \geq 0} \gg 0$, with $r(t)$ being the interest rate and $w(t)$ the labor wage per unit of time at each instant, the budget constraint faced by household $i \in \mathcal{I}$ is,

$$\dot{a}^i(t) = r(t) a^i(t) + \theta^i(t) w(t) - c^i(t) , \quad (3)$$

for all $t \geq 0$, ($\dot{x}(t) \equiv dx(t)/dt$ for any variable x) and the transversality condition is,

$$\lim_{t \rightarrow \infty} e^{-\int_0^t r(\tau) d\tau} a^i(t) = 0 . \quad (4)$$

We define the domains of wealth- and productivity heterogeneity at any given price vector, for which the existence of a representative consumer is conceptually relevant. That is the domain that guarantees interiority of solutions to each individual optimization problem. The following assumption states this formally.

Assumption 3 *Given a community preference profile captured by the collection of functions $(u^i)_{i \in \mathcal{I}}$ and ρ , the domain of, (i) initial distribution of assets $(a_0^i)_{i \in \mathcal{I}}$, (ii) the collection of labor-productivity functions $(\theta^i)_{i \in \mathcal{I}}$, and (iii) prices $(r(t), w(t))_{t \geq 0}$, is restricted so that the optimization problems of all households $i \in \mathcal{I}$ are well-defined, and the solution to each individual problem is interior for all $t \geq 0$.*

Given Assumption 3, maximizing (2) subject to constraints (3) and (4) for any given a_0^i is an optimal-control problem with necessary optimality conditions given by,

$$\dot{c}^i(t) = -\frac{u_1^i(c^i(t))}{u_{11}^i(c^i(t))} [r(t) - \rho(t)] , \quad (5)$$

together with (3) and (4), that lead to decision rules of the form,

$$c^i(t) = C^i \left(a^i(t), t \mid (r(\tau), w(\tau), \theta^i(\tau))_{\tau \geq t} \right), \quad (6)$$

i.e., consumption rules at each moment are memoryless, depending only on current personal assets and current and future prices. Assumptions 1 and 3 have a particular connection, that is revealed from equation (5). The term $-\frac{u_1^i(c^i(t))}{u_{11}^i(c^i(t))}$ must always be well-defined in order to have interiority. Thus, to meet Assumption 3 (interior solutions), it is necessary that $c^i(t) \in \mathbb{C}^i$, for all $t \geq 0$, and all $i \in \mathcal{I}$.

Definition 1 *Given a community preference profile captured by the collection of functions $(u^i)_{i \in \mathcal{I}}$, and ρ , complying with Assumptions 1 and 2, a representative consumer (denoted by “RC”) is a (fictitious) consumer who has time-separable preferences, $\int_0^\infty v^{RC}(c(t), t) dt$, with $v_1^{RC}(c, t)$, $v_{11}^{RC}(c, t)$ and $v_{12}^{RC}(c, t)$ existing, and with $0 < v_1^{RC}(c, t) < \infty$ and $-\infty < v_{11}^{RC}(c, t), v_{12}^{RC}(c, t)$ well-defined for all consumption levels, $c \in \mathbb{C}^{RC} \equiv \{c \in \mathbb{R}_+ \mid c = \int_{\mathcal{I}} c^i d\mu(i), c^i \in \mathbb{C}^i, i \in \mathcal{I}\}$, for all $t \geq 0$, and who possesses the economy-wide aggregate wealth and productivity at all times, and whose demand functions coincide with the aggregate demand functions of the economy at all times, namely,*

$$\begin{aligned} c^{RC}(t) &= C^{RC} \left(\int_{\mathcal{I}} a^i(t) d\mu(i), t \mid \left(r(\tau), w(\tau), \int_{\mathcal{I}} \theta^i(\tau) d\mu(i) \right)_{\tau \geq t} \right) = \\ &= \int_{\mathcal{I}} C^i \left(a^i(t), t \mid (r(\tau), w(\tau), \theta^i(\tau))_{\tau \geq t} \right) d\mu(i), \quad (7) \end{aligned}$$

for all $t \geq 0$, for the complete domain of prices $(r(t), w(t))_{t \geq 0}$, initial distributions of assets, $(a_0^i)_{i \in \mathcal{I}}$, and functions $(\theta^i : \mathbb{R}_+ \rightarrow \mathbb{R})_{i \in \mathcal{I}}$ that comply with

Assumption 3.

This is a rather strong representative-consumer concept: it focuses on solving only one household’s problem using standard optimal-control techniques, in order to derive aggregate demands at all times.¹⁴ Our goal is to examine conditions on the community preference profile that are necessary and sufficient for the existence of social preferences (representative-consumer preferences) consistent with the independence axiom of Koopmans (1960): if two different intertemporal paths have a common outcome at a certain point in time, preferences over these two paths should always, and solely, be determined by comparing them with remaining outcomes at that particular date that differ. In other words, the focus of our analysis is to characterize community preference profiles where social preferences are time-separable and, at each separate point in time, non-intersecting social indifference curves exist.

Assumption 4 $\bigcap_{i \in \mathcal{I}} \mathbb{C}^i$ is non-empty and not a singleton.

Assumption 4 places a weak constraint on the scope of preference heterogeneity. It says that nobody’s bliss point (if any), should be lower than or equal to anyone else’s subsistence level of consumption (if any), hence $\bigcap_{i \in \mathcal{I}} \mathbb{C}^i$ is an interval. Since the consumable good is considered to be a composite good (a consumer basket), Assumption 4 is not unreasonably restrictive.

Theorem 1 *Under Assumptions 1 through 4, a representative consumer exists if and only if*

¹⁴Our aggregation concept differs from the aggregation concept used by Gollier and Zeckhauser (2005) who use Pareto weights to construct the objective of a “representative agent”.

$$u^i(c) = \begin{cases} \frac{(\alpha c + \beta_i)^{1-\frac{1}{\alpha}} - 1}{\alpha(1-\frac{1}{\alpha})} & \text{with } \alpha > 0 \text{ and } \beta_i \in \mathbb{R} \text{ or } \alpha < 0 \text{ and } \beta_i > 0 \\ \text{or} & \\ -e^{-\frac{1}{\beta_i}c} & \text{with } \beta_i > 0 \end{cases}, \quad (8)$$

for all $i \in \mathcal{I}$. The representative consumer has the common, across households, rate of time preference, $\rho(t)$, at all times, and momentary utility function given by,

$$u^{RC}(c) = \begin{cases} \frac{(\alpha c + \beta_{RC})^{1-\frac{1}{\alpha}} - 1}{\alpha(1-\frac{1}{\alpha})} & \text{for } \alpha \neq 0 \\ -e^{-\frac{1}{\beta_{RC}}c} & \text{else} \end{cases}, \quad (9)$$

with

$$\beta_{RC} = \int_{\mathcal{I}} \beta_i d\mu(i) .$$

Proof Theorem 1 See Appendix “Proofs”. \square

Theorem 1 states comprehensively that the existence of a representative consumer rests upon particular functional forms and common parameter values: the quasi elasticity of intertemporal substitution, α , should be the same across all households;¹⁵ households can differ only with respect to their subsistence consumption or bliss point of consumption; yet, it is either that all households have some subsistence consumption, or that all households have some bliss point, but bliss points and consumption subsistence levels cannot coexist in the same community preference profile. This category of utility functions is the identical-shape absolute-risk aversion (ISHARA) utility class. Mazzocco (2007) has provided a similar result to Theorem 1. Yet, Mazzocco’s (2007) is undertaken in discrete time, relying on previous

¹⁵Notice that the elasticity of substitution is equal to $\alpha \cdot (1 - \beta_i/c)$.

proofs provided by Pollak (1971). Our analysis here is undertaken in continuous time, which requires a different approach for establishing and proving Theorem 1. Our continuous-time approach is most useful for extending our analysis to introducing heterogeneous rates of time preference.

These restrictions on the community preference profile, $(u^i)_{i \in \mathcal{I}}$, lead to *common orientation of incentives and actions of rich and poor, or large versus small, households*. In particular, the *consumption decision rules* of all household types, $i \in \mathcal{I}$, are of the form,

$$c^i(t) = b(t) a^i(t) + \zeta^i(t) ,$$

i.e., they *are always linear in financial wealth, $a^i(t)$, and parallel across all households*.¹⁶

Yet, in order to accommodate the idea that households may switch type over time, in the next section we explore the case where momentary utility is time-variant.

2.2 Time-Variant Momentary Utility Functions: heterogeneous rates of time preference and the extended ISHARA utility class

We examine the case where individual rates of time preference have a consumption-choice-independent part which is common-across households, and a consumption-choice-dependent part implied by their momentary utility function.¹⁷ In particular, consumer preferences of each $i \in \mathcal{I}$, are given by the general additively-separable utility function, which implies

¹⁶For this linearity in decision rules see the sufficiency part in the proof of Theorem 1 in Appendix “Proofs”. Carroll and Kimball (1996) show that if labor income uncertainty is introduced, the consumption function becomes concave. Nevertheless, the class of preferences we have identified promotes a linear shape for the consumption function, at least when wealth is sufficiently far from borrowing constraints.

¹⁷Koulovatianos (2005, Theorem 2) provides an analysis for a class of utility functions that allows for heterogeneous choice-independent rates of time preference. The necessary class of utility functions allowing for linear aggregation is very restricted: it is utility functions with constant absolute risk aversion.

heterogeneous rates of time preference,¹⁸

$$U^i \left((c^i(t))_{t \geq 0} \right) = \int_0^\infty e^{-\int_0^t \rho(\tau) d\tau} u^i(c^i(t), t) dt . \quad (11)$$

with $\rho : \mathbb{R}_+ \rightarrow \mathbb{R}_{++}$. A sequence of assumptions are important for the analysis that follows.

Assumption 5 For all $i \in \mathcal{I}$, and all $t \geq 0$, $u^i : \mathbb{R}_+^2 \rightarrow \mathbb{R}$, is twice-continuously differentiable with respect to c , once continuously differentiable with respect to t , and such that $u_1^i(c, t) > 0$ and $u_{11}^i(c) < 0$ on some interval, $\mathbb{C}^i(t) \subseteq \mathbb{R}_+$, with $u_1^i(c) < \infty$, $-\infty < u_{11}^i(c)$, $-\infty < u_{12}^i(c) < \infty$ for all $c \in \mathbb{C}^i(t) \subseteq \mathbb{R}_+$, with $\underline{c}^i(t) \equiv \inf(\mathbb{C}^i(t)) < \sup(\mathbb{C}^i(t)) \equiv \bar{c}^i(t)$.

Assumption 6 For all $i \in \mathcal{I}$, $\bigcap_{t \geq 0} u_1^i(\mathbb{C}^i(t), t)$ is non-empty and not a singleton.

Assumption 7 For any $i \in \mathcal{I}$, let,

$$\mathfrak{C}^i \equiv \left\{ c \in \bigcap_{t \geq 0} \mathbb{C}^i(t) \mid \bigcap_{t \geq 0} u_1^i(\mathbb{C}^i(t), t) \text{ is non-empty and not a singleton} \right\} .$$

Then, $\bigcap_{i \in \mathcal{I}} \mathfrak{C}^i$ is non-empty and not a singleton.

Based on these assumptions, Theorem 2 states an aggregation result that pertains to this setting.

¹⁸In equation (11), the functional form $\int_0^\infty e^{-\int_0^t \rho(\tau) d\tau} u^i(c^i(t), t) dt$ could have also been expressed as $\int_0^\infty v^i(c^i(t), t) dt$. We have not expressed momentary utility as $v^i(c^i(t), t)$, because at any $t \geq 0$ we want to the rate of time preference to be equal to,

$$-\frac{v_{12}^i(c^i(t), t)}{v_1^i(c^i(t), t)} = \rho(t) - \frac{u_{12}^i(c^i(t), t)}{u_1^i(c^i(t), t)} . \quad (10)$$

The above equation shows that the rate of time preference is additively separable in a choice-independent ($\rho(t)$) and a choice-dependent component ($-u_{12}^i/u_1^i$). This formulation offers convenience in constructing asymptotically convergent paths of the present value of consumption in our proofs. Notice also that the rates of time preference are a-priori heterogeneous.

Theorem 2 *Under Assumptions 2, 3, and 5 through 7, a representative consumer exists if and only if*

$$u^i(c, t) = \begin{cases} \frac{[\alpha c + \beta^i(t)]^{1-\frac{1}{\alpha}} - 1}{\alpha(1-\frac{1}{\alpha})} & \text{with } \alpha > 0 \text{ and } \beta^i(t) \in \mathbb{R} \text{ or } \alpha < 0 \text{ and } \beta^i(t) > 0 \\ \text{or} \\ -e^{-\frac{1}{\beta_i G(t)} c} & \text{with } \beta_i > 0 \end{cases}, \quad (12)$$

for all $i \in \mathcal{I}$, with functions $\beta^i(t)$ such that Assumptions 6 and 7 are met.

The representative consumer has

$$U^{RC}((c(t))_{t \geq 0}) = \int_0^\infty e^{-\int_0^t \rho(\tau) d\tau} u^{RC}(c(t), t) dt, \quad (13)$$

with,

$$u^{RC}(c, t) = \begin{cases} \frac{[\alpha c + \beta^{RC}(t)]^{1-\frac{1}{\alpha}} - 1}{\alpha(1-\frac{1}{\alpha})} & \text{for } \alpha \neq 0, \beta^{RC}(t) = \int_{\mathcal{I}} \beta^i(t) d\mu(i) \\ -e^{-\frac{1}{\beta_{RC} G(t)} c} & \text{else, } \beta_{RC} = \int_{\mathcal{I}} \beta_i d\mu(i) \end{cases}. \quad (14)$$

Proof Theorem 2 See Appendix “Proofs”. \square

The message conveyed by Theorem 2 and the additional insight to Theorem 1 is that parameter α is not only common across all household types, but also α cannot vary over time. The only utility parameters that can vary over time are subsistence levels (or bliss points) $\beta^i(t)$. As long as $\beta^i(t)$ for some $i \in \mathcal{I}$, preferences given by (12) imply that both rates of time preference and elasticities of intertemporal substitution are heterogeneous, yet, exact linear aggregation applies. While Theorem 2 is precise about the functional forms that allow for aggregation, it does not offer a precise characterization of the trajectories of parameters $\left((\beta^i(t))_{t \geq 0} \right)_{i \in \mathcal{I}}$. The trajectories of these parameters are free, subject to our

interiority assumptions regarding each household's optimization problem. Notice also that if utility functions are of the exponential form, then $\beta^i(t)/\beta^i(0) = \beta^j(t)/\beta^j(0)$ for all $i, j \in \mathcal{I}$, a restriction that does not hold for all other utility functions of Theorem 2.

As rates of time preference are given by equation (10), they can be heterogeneous. If $\beta^i(t) \neq 0$, then elasticities of intertemporal substitution, $\alpha \cdot [1 - \beta_i(t)/c]$, can be heterogeneous, too. These two aspects of heterogeneity are in line with a large body of heterogeneous-agent literature in macroeconomics and household finance. These features characterize the extended ISHARA utility class identified by Theorem 2.

2.2.1 How static and dynamic aggregation analyses differ

The distinction between static and dynamic aggregation analysis has implications for the empirical testing of aggregation. Static-model demand aggregation coincides with linear and parallel Engel curves. The consumption decision in a static model (regarding a consumer-item category, in our model we have the comprehensive consumer basket since there is a savings option) would be of the form $c^i = b_0^i + b_1 y^i$, and parallel Engel curves would require that b_1 is common across households while b_0^i would be allowed to vary across households. Here we do not have just Engel curves but decision rules involving wealth and future income anticipations. Our decision rules are of the form (see explicit formulas (61) and (62) in the Appendix),

$$c^i(t) = b_0^i \left(t \mid (\beta^i(\tau))_{\tau \geq t} \right) + b_1(t) y^i(t) + b_2(t) a^i(t) + b_3^i \left(t \mid (y^i(\tau))_{\tau > t} \right). \quad (15)$$

The decision rule given by (15) is linear in $y^i(t)$ and with common-across-households coefficient $b_1(t)$ at time t (notice that $b_1(t)$ can also vary over time). But it includes household wealth, a variable most often missing from household microdatabases and it also contains two time-varying coefficients, $b_0^i \left(t \mid (\beta^i(\tau))_{\tau \geq t} \right)$ and $b_3^i \left(t \mid (y^i(\tau))_{\tau > t} \right)$. The latter depends on

anticipation of future income and it is different across individuals, the former depends on anticipation of changes in household preferences, e.g., due to anticipated changes in household size. Failing to sufficiently control for wealth, life-cycle effects, future-income anticipations and anticipations about future taste changes due to demographic heterogeneity, may lead to an erroneous rejection of actual linearity between consumption categories and current income. Interestingly, in a dynamic empirical analysis, Calvet and Comon (2003) report evidence that failing to account for taste heterogeneity can lead to imprecise conclusions about relationships between budget shares and income.

2.3 Application to Household-Size Heterogeneity and the Necessity of the Linear Relationship Across EIs

If individuals have utility from the class presented in Theorems 1 and 2, then these two theorems imply that whenever such individuals form multi-member households, the objective function of household members can be merged into a single utility function of the representative household member. That utility function will be of the same functional form but with a different parameter $\beta^i(t)$, and rescaled to capture the change in household size.

Assume that $\rho(t) = \rho$ for all t , another standard assumption in the literature, and also that households of the same size all have the same utility function. If a representative consumer exists, then utility functions should fall in the class given by Theorem 1. Focusing on the case where $r(t) = \bar{r} = \rho$ for all t , and with $w(t) = \bar{w}$, and $\theta^i(t) = \bar{\theta}^i$ for all t , a steady-state condition for all households, (5) and (3) imply that

$$\bar{c}^i = \bar{r}a^i + \bar{w}\bar{\theta}^i = \bar{y}^i, \quad (16)$$

where \bar{y}^i is the permanent income of household i in the steady state.¹⁹ Moreover, we assume

¹⁹In the empirical analysis below it can be seen that the way we have designed our survey questionnaire, asking respondents about monthly incomes, approximates the above steady state conditions where households consume their incomes.

that each household type stays the same forever, with $\beta^i(t) = \beta_i$, for all $t \geq 0$, when preferences of Theorem 2 apply. So, the model referring to Theorem 2 above collapses to the model of Theorem 1.²⁰ On such a permanent income trajectory, the lifetime utility of a household is $\int_0^\infty e^{-\rho t} u^i(\bar{y}^i) dt = u^i(\bar{y}^i) / \rho$. We define a set of *permanent equivalent incomes*, denoted by \bar{y}_E , of households belonging to any two different family types $i, j \in \mathcal{I}$, as incomes that equate lifetime utilities, $u^i(\bar{y}^i) / \rho = u^j(\bar{y}^j) / \rho$, i.e. any pair $(\bar{y}_E^i, \bar{y}_E^j)$, $i, j \in \mathcal{I}$, that solves

$$u^i(\bar{y}_E^i) = u^j(\bar{y}_E^j) . \quad (17)$$

In Proposition 1 which appears below we present a result that holds under the above steady-state conditions. In Section 3 we empirically implement a survey instrument that mimics such a stringently controlled environment.²¹ So, the empirical relevance of the restrictive assumptions made in Proposition 1 is that our survey instrument is capable of controlling hypothetical situations of households, mimicking and approximating steady state conditions where households consume their incomes.

Proposition 1 *Let all households of the same size have the same utility function.*

If $r(t) = \bar{r} = \rho$ for all $t \geq 0$, and all household types receive their permanent incomes \bar{y}^i for all $i \in \mathcal{I}$. Then if each household type stays the same forever, with $\beta^i(t) = \beta_i$, for all $t \geq 0$, the existence of a representative consumer implies that for all $i, j \in \mathcal{I}$,

$$\bar{y}_E^j = \chi_{i,j} + \psi_{i,j} \bar{y}_E^i , \quad (18)$$

for some $\chi_{i,j} \in \mathbb{R}$, and $\psi_{i,j} > 0$.

²⁰The interpretation of the model of Theorem 2 is the case where a dynasty is shifting from one household type to another over time, keeping smoothness of such transitions so as to comply with Assumption 5.

²¹In particular, we ask our respondents questions about evaluating the material comfort of individuals living under hypothetical exogenously fixed conditions: individuals who live in households with particular demographic composition and who receive constant levels of monthly household income streams.

Proof See Appendix “Proofs”. \square

In words, equation (18) means that if all household types are in a zero-growth steady state where they receive their permanent income, then *household size-economies entailed in discretionary incomes (income minus subsistence needs) are the same across the rich and the poor.*²² Corollary 1 is an immediate consequence of Proposition 1, that we state without proof.

Corollary 1 *Empirically falsifying equation (18), falsifies the existence of a representative consumer.*

Corollary 1 shows that testing the existence of the whole representative-consumer construct in the strict linear-aggregation sense invented by Gorman (1953), can collapse through the testing of a single equation, equation (18). Throughout the rest of this paper we claim that equation (18) is testable, we demonstrate how it can be tested, and we put it under scrutiny.

3. Empirical Analysis

As shown in Proposition 1, i.e., equation (18), the existence of a representative consumer requires that household-size economies entailed in permanent equivalent incomes, after sub-

²²We remind that in this case where households are in a zero-growth steady state ($r(t) = \bar{r} = \rho$), and receiving their permanent income, households optimally choose to spend their discretionary income at every instant of time. For some ξ , equation (18) can be re-written as

$$y^j - \xi \cdot \chi_{i,j} = \psi_{i,j} \left(y^i - \frac{\xi - 1}{\psi_{i,j}} \chi_{i,j} \right) ,$$

where ξ is to be identified empirically, and it will specify subsistence needs for the two types. In our empirical analysis in the next section we find that, if i in the equation right above is a one-member household, then $\chi_{i,j} > 0$ in all cases. That $\chi_{i,j} > 0$ in all cases where j is a multi-member household implies that there are subsistence needs which increase as household members are added in the household, and it motivates that $\xi > 1$ (each household has non-trivial subsistence needs). In the above equation $\psi_{i,j}$ measures the economies of household size achieved by discretionary incomes across household types i and j , and the equation implies that these are the same for the rich and the poor.

tracting subsistence needs, are the same across the rich and the poor. Empirically testing (18) requires data on EIs. Identifying EIs from observed choices, i.e. demand data, has occupied researchers since long ago.²³ Most researchers use demand systems in order to impose a theoretical framework for identifying EIs. Their works share a common feature: identification assumptions must be imposed. Examples for such identification assumptions are “independence of base,” or “generalized (absolute) equivalence scale exactness”. Imposing an identification assumption means to postulate a particular relationship for EI functions. For example, Lewbel (1989) and Blackorby and Donaldson (1993) assume a special case of (18) where $\chi_{i,j} = 0$, in order to identify EIs, while Donaldson and Pendakur (2006) impose (18) in their estimation. Yet, these assumptions are a-priori untestable. For these reasons we have designed a survey method where we ask respondents to provide their own assessments of EIs for a set of household types. Specifically, we ask respondents: “What is the net monthly household income that can make a household with two adults and a child attain the same level of material comfort as that of a one-member household with a net monthly income of \$2,000?”²⁴

Here we employ data from our survey method only, without combining them with observed consumer choices. A key reason is that, in order to test for the validity of (18) which refers to households receiving their permanent income continuously, it would be very difficult to isolate such real-world households in observed data. Through the design of our survey method, we use our respondents as our control device for implementing such

²³A key objective in inequality measurement is to utilize available income data at the household level in order to construct measures of incomes that are one-member-household equivalent. In this way the distribution of living standards across *individuals* can be measured from an observed distribution of household incomes.
²⁴Another important aspect of approaching this demanding estimation task is that the validity of equation (18) requires to distinguish experiences where a household receives its permanent income every period. Identifying such a special intertemporal income profile with constant income receipts in actual data is difficult and it restricts possibilities of performing this test. Our survey instrument serves as a control device where respondents are asked to perform a particular thought experiment that replicates the income profile behind equation (18).

permanent-income situations, by asking them to perform thought experiments about hypothetical households. Another reason we do not employ observed consumer-choice data is that we are not interested in parameter estimates; instead we are only interested in performing a specification test of the functional form given by (18).

The motivation of our survey relies on the idea that respondents are experienced at recognizing the connection between a household’s demographic composition and the level of material comfort that income can buy for its members. In this sense, respondents are ‘real-life experts’ in assessing EIs. Pooling diverse insights of a large number of respondents may correct potential biases of a single expert. Yet, respondents must have sufficient information to assess EIs for households with a demographic composition and a level of material comfort that differ from their own actual experiences. Otherwise, estimates of EI may suffer from limited information bias (LIB). In order to test for LIB we use a large sample that is representative with respect to the income dimension and oversamples household types that are scarce in the overall population (for example, single parents with two or more children). Moreover, respondents should demonstrate sufficient understanding in answering the question about assessing EIs. To test for this crucial aspect of survey effectiveness, we also pose an equivalent assessment problem using different means of representation, and then cross-check for consistency. So, our survey instrument is equipped with a tool that tests whether people ‘mean what they say’.²⁵

3.1 Overview of Survey Design

Our questionnaire consists of two main parts.²⁶ In Part A, we pre-assign a net monthly income for a one-member household, a reference income (RI), and ask respondents to state

²⁵See, for example, Bertrand and Mullainathan (2001) for a discussion of the validity of respondent data in surveys.

²⁶Our questionnaire appears in Appendix E.

EIs for seven other household types. Each respondent is randomly assigned one of five different RIs. The question asked is of the following type: “What is the net monthly household income that can make a household with two adults and a child attain the same level of material comfort as that of a one-member household with a net monthly income of \$2,000? What income would one need if, instead, there were two children in the household?”²⁷ We emphasize that we use *material comfort* or living standard, as measured by the goods, services, and luxuries available to an individual or group, which should be distinguished from the concept of overall life satisfaction used in Kahneman and Krueger (2006) and Kahneman et al. (2006).

In Part B we pose an equivalent assessment problem to this of Part A, using different means of representation to cross-check for consistency: Likert-scale evaluations of material comfort.²⁸ The question we ask is: “Consider that the net monthly household income of a household with two adults and one child is \$5,500. State a number from 1 to 100 that best characterizes the level of material comfort of this household, given that ‘10’ is ‘very bad,’ ‘50’ is ‘sufficient,’ and ‘90’ is ‘very good.’” Respondents receive such a question for the one-member household and the seven household types of Part A. Household incomes evaluated in Part B were obtained through a previous pilot study in Germany using the same RIs as in Part A.²⁹ If a respondent states a Likert-scale value for a household type with pre-assigned income Y that is higher than what she/he stated for the one-member household with the RI in Part B, then, in Part A, this respondent should have stated an EI for that household

²⁷A crucial aspect in the design of Part A is which RIs to choose. In Appendix A we explain in detail how we have calibrated the five different levels of RI combining information from the actual German income distribution of one-member households and information about social benefits for the poorest households in Germany. In our surveys we have calibrated RIs by picking five incomes that span the range from the 5th up to the 95th percentile of the one-member-household distribution of disposable income and split it in four equi-spaced segments.

²⁸For an example of Likert-scale evaluations see Kahneman et al. (2004).

²⁹The previous pilot study is the German data appearing in Koulovatianos et al. (2005).

type that is lower than Y .

3.2 Survey Samples

In our empirical analysis we use a large sample of 2,042 respondents from Germany collected in year 2006. Details about data collection and personal characteristics of respondents appear in Appendix A. This large sample is from all regions of Germany and it is representatively sampled along the dimension of household incomes. Yet, in order to secure sufficient power of LIB tests, we have over-sampled single parents with two or more children, and all relevant details also appear in Appendix A. The intended over-representation of respondents having children has contributed considerably to the high percentage of female respondents.

Previously to conducting this large German survey we have run six pilot studies in countries as different as Botswana, China, Cyprus, France, Germany, and India. There is a key difference in survey design between the large German survey and the six pilot surveys. In the large German survey, in Part A, each respondent was asked to provide EIs for seven household types, all referring to the a single randomly selected RI (out of five available RIs). In the six pilot studies, each respondent has been asked to do the same, but for all five RIs, i.e., each respondent has provided 35 EI assessments in total.³⁰ This is a reason why our small pilot samples (ranging from 130 to 223 respondents) are sufficient. In addition, the pilot surveys do not include part B of the questionnaire.

3.3 What the Raw Data Say

The scatter plots of uncontrolled (raw) responses in Part A of the survey appear in Figure 1. Each panel refers to a household type distinguished by demographic composition. On the horizontal axis of each panel is the one-member household income (RI), five fixed levels

³⁰The only exception is Botswana, where three instead of five RIs have been provided (see Appendix B for details).

that are exogenously determined in the questionnaire (amounts are in 2006 Euros). Against these fixed RIs we plot the survey responses about EIs. In each panel there are 2,042 scatter points, each corresponding to a response by each of our 2,042 respondents. So, in each panel there is one EI assessment by each respondent. Crucial for the test of the affine relationship given by (18) is that *in each panel of Figure 1, the group of respondents corresponding to each RI (each vertical scatter) is independent (disjoint) from all other respondent groups that stated EI assessments for different RIs (about 400 respondents for each RI).*

Each panel of Figure 1 suggests that the relationship between EI and RI is affine: for all seven household types, a sixth-degree polynomial least-squares curve (solid line) is hardly distinguishable from a linear fit (dashed line). Only for the fourth RI (EUR 2,750), the polynomial fit indicates a slight deviation downwards. In brief, *Figure 1 suggests that five independent groups of respondents seem to place their assessments of EIs on a straight line for seven different household types, on average.*

The affine relationship among EIs is also present in all pilot studies appearing in Figures 2 and 3 (raw responses again). The structure of Figures 2 and 3 is the same as this in Figure 1, with the sole difference that we have merged scatter plots from three countries in each figure and that we present only sixth-degree polynomial fits (and not linear fits as it is the case with the dashed lines appearing in Figure 1). For inter-country comparisons all amounts appearing in Figures 2 and 3 are in purchasing-power-parity (PPP) adjusted Euros for Germany in year 2006. In all panels of Figures 2 and 3 the sixth-degree polynomial fit is visually close to a line. Nevertheless, we remind that in our pilot studies, each respondent provided assessments for all five RIs for each household type. So, in each panel of Figures 2 and 3 one respondent (per country) has provided five scatter points (EI assessments), one for each RI.

Regarding our pilot surveys, an objection can arise concerning the validity of our tests about affine relationship (18). If each respondent follows an affine rule of thumb to provide EI assessments, the average picture in each country could have been a result of framing: respondents may be lazy to think and perhaps follow an affine rule of thumb that dominates the total picture. For this reason, in the large-scale German survey each respondent assesses EIs for a single (randomly assigned) RI. Further, the large-scale survey collects data that allow an assessment of whether respondents mean what they say. We devote the remainder of this section to providing formal tests of (18), and for the effectiveness of our survey method.

3.4 Regression Analysis

3.4.1 Overview of Goals

Our empirical analysis has four goals. First, we perform a specification test for the affine relationship given by (18). We assume (18) as candidate specification in household-type specific regression models where stated EIs are the endogenous variable and RIs are the exogenous variable. We also assign additional dummy variables for RIs, so as to test if there is any variation left unexplained by the affine relationship given by (18).

Our second goal is to test whether respondents “mean what they say”, i.e. if they understand the context of the questions correctly. As we have explained above, in Parts A and B of the questionnaire we have provided the same assessment problem using different means of representation. So, we construct a variable capable of cross-checking for consistency between responses of the same respondent in Parts A and B.

Our third goal is to test whether the rich sufficiently understand the needs of the poor and whether the poor understand the wants of the rich, i.e., whether respondents understand the determinants of material comfort of individuals living in family types different from the respondent’s family type. If this understanding is limited, then responses may suffer from

LIB. To test for LIB, answers from respondents who state an EI for the household type and/or living standard that is the same as their own, are distinguished from answers where this is not the case. The presence of LIB is tested in regression analysis through a test of exclusion of dummy variables that identify this relationship between respondents' personal characteristics and the features of households that respondents evaluate.

Our fourth goal is to test whether personal characteristics of respondents affect their assessments. So, in regression analysis we use a large set of personal characteristics of the respondents as conditioning variables and test for their potential impact. Since sampling is representative in the large sample for Germany, these tests should have sufficient statistical power.

3.4.2 Regression Model and Results

Our regression model for the large survey from Germany is,

$$\begin{aligned}
 ES_i^h = & f^h(RI_i) + b_0^h RI_Dummies_i + b_1^h NLSE_i^h + b_2^h LIB_{h,i} + b_3^h LIB_{mc,i} \\
 & + b_4^h (LIB_{h,i} \cdot LIB_{mc,i}) + b_5^h Personal_Characteristics_i + \varepsilon_i^h, \quad (19)
 \end{aligned}$$

where

$$f^h(RI) = a^h + \frac{b^h}{RI}. \quad (20)$$

The dependent variable is defined as $ES_i^h = EI_i^h / RI_i$, where EI_i^h is the EI stated by respondent i about household type h , given that respondent i was asked to state EIs using a one-member household with RI equal to RI_i as a benchmark. Because an EI divided by RI is an Equivalence Scale (ES), ES_i^h is i 's assessment of the ES concerning household type h , given the RI level that was assigned to i in Part A of the questionnaire.

The function $f^h(RI_i)$ given by (20) in equation (19) is a proposed candidate for offering an accurate explanation of the relationship between RIs and ESs and complies with (18).

The term ε_i^h is the error term. Definitions and roles of all conditioning variables in equation (19) appear below.

Variable $RI_Dummies_i$: testing the affine relationship (18). This is a set that can include up to three dummy variables related to RI_i , the RI assigned to respondent i in Part A.³¹ If, for example, the RI equal to EUR 2,000 is included in this set, then the $RI_Dummy(=EUR\ 2,000)$ takes the value of 1 for all respondents who were assigned RI equal to 2,000 EUR, and 0 otherwise. The conditioning set $RI_Dummies_i$ is the instrument for conducting the specification test for any candidate function $f^h(RI)$: if there is any variation left unexplained by the affine relationship (18) that is now transformed into (20) in regression (19), then it should be captured by $RI_Dummies_i$; so a test of exclusion of $RI_Dummies_i$ reveals whether $f^h(RI)$ satisfactorily captures the dependence of ESs on RI. In Table 1 most of these RI dummy variables are insignificant. Only the RI dummy variable at RI = EUR 2,750 is significant (based on t -tests), but it suggests only a small deviation from the affine relationship (18). The exclusion tests concerning all three RI dummy variables have moderately low F-test statistics. *None of these tests rejects exclusion with a confidence level of 99% or more.* In sum, given how tough this test of exclusion is, equation (20) gives a reasonable specification for $f^h(RI)$, which has meaningful intuition. Coefficient b^h in (20) can be interpreted as fixed costs in consumption, *in addition* to the fixed costs of the one-member household. The constant a^h in equation (20) is a measure of household-size economies after controlling for the presence of household-type specific fixed costs in consumption. As household income increases, fixed costs become a smaller share of a household's budget. In other words, ES is a *decreasing* function of RI.

Variable $NLSE_i^h$: testing whether respondents “mean what they say”. The acronym

³¹The set $RI_Dummies_i$ can contain up to three RI dummy variables, since four RI dummy variables together with a constant (a^h) are perfectly correlated with the monotonic function b^h/RI .

NLSE stands for “Normalized Likert Scale Evaluation”, and the NLSE value of respondent i for a household type h is given by $NLSE_i^h = \ln(L_i^h/L_i^*)$, where L_i^h denotes respondent i 's stated Likert-scale value for household type h , and L_i^* denotes the Likert-scale value *given by the same respondent, i* , for the one-member household in Part B of the questionnaire. Each respondent was provided with only one RI to evaluate in Part B, again randomly assigned.³² The NLSE uses the stated Likert-scale value concerning the one-member household as a benchmark, and measures the deviation of each other Likert-scale value stated by the same respondent from this benchmark. In Appendix D we provide evidence that the NLSE is effective in suppressing noise from *Heterogeneity in Respondent Perceptions of Verbal Characterizations*.³³ If the sign of b_1^h in regression (19) is negative, then a necessary condition behind the hypothesis that respondents understand the main evaluation task in Part A finds affirming evidence. Moreover, the estimator of b_1^h may control for some respondents' deviant opinions about, e.g., the cost of partners or children, so a test of exclusion of the NLSE in the regression provides information about the possible presence of such deviant evaluations.³⁴ In Table 1 we can see that all NLSE coefficients have a negative sign and all tests

³²The RIs in Part A are assigned independently from those assigned in Part B. This feature of the survey design helps to avoid the possibility that the NLSE is spuriously correlated with the dependent variable in the regression analysis appearing in Table 1 in the text. Spurious correlation may result from having the same respondent focusing on the same level of material comfort in the evaluations of Parts A and B: some respondents may consciously attempt to provide consistent responses between Parts A and B, instead of focusing on the evaluation question in each Part.

³³Heterogeneity in the way respondents perceive words is discussed in Appendix C. Kahneman and Krueger (2006, pp. 19-21) discuss this issue and propose a technique for coping with this problem in their analysis that uses Likert scales on verbal descriptions of well-being (although, as we have explained above, the concept of well being we use in our survey is material comfort instead of “happiness”). Similar concerns are also discussed in Bertrand and Mullainathan (2001). In Appendix C we present evidence on how our NLSE variable deals with this problem of stated Likert scale evaluations by respondents.

³⁴See Pollak and Wales (1979) for concerns about consumer choices and fertility preferences. For example, biases stemming from any possible, say, dislike about children by respondents may be corrected by the inclusion of NLSE, which offers a way to deal with the critique by Pollak and Wales (1979) about “conditional” vs. “unconditional” equivalence scales. In Koulovatianos et al. (2005, p. 990) we have discussed that we do not expect our survey method to be influenced strongly by the conceptual distinction raised by Pollak and Wales (1979). The mild role played by the inclusion of NLSE in the regressions (see the quantitatively low NLSE coefficients in Table 1) reconfirms our earlier conjecture in Koulovatianos et al. (2005, p. 990).

of exclusion are rejected ($P < 0.001$). These findings support the effectiveness of the survey method. Moreover, the size of all NLSE coefficients is small, indicating that respondents' deviant opinions about household-size economies do not affect the estimators of coefficients of equation (20) to a large extent.

$LIB_{h,i}$, $LIB_{mc,i}$, ($LIB_{h,i} \cdot LIB_{mc,i}$): *testing whether respondents understand the living standards of households with features different from their own.* In order to enable tests of LIB that have sufficient statistical power, the sampling strategy should ensure that there are enough respondents who live in each of the household types that appear as hypothetical households in Part A.³⁵

Let respondent i belong to household type h and let Y_i be the disposable household income of respondent i . From responses to Part A, we calculate five average EIs for household type h , each corresponding to an RI. We identify the average EI for household type h that is closest to Y_i . This identified average EI corresponds to an RI that should give the same level of material comfort for the one-member household. If this particular RI coincides with the RI that was randomly assigned to i in Part A, then i performed the MET for hypothetical households with material comforts close to his/her own. We use this identification procedure to create the dummy variables,

$LIB_{mc,i} = 1$ if respondent i 's material comfort is closest to the material comfort of the one-member household, based on the RI that respondent i evaluated in Part A; 0 otherwise; and

$LIB_{h,i} = 1$ if respondent i belongs to household type h , and the dependent variable in the regression refers to household type h ; 0 otherwise.

³⁵Since the RIs chosen in Part A cover a wide range of one-member-household disposable incomes, sampling should be such that, for each household type, the respondents' household income represents a wide range of the economy's household incomes.

Variables $LIB_{h,i}$, $LIB_{mc,i}$, and the product $LIB_{h,i} \cdot LIB_{mc,i}$, serve as conditioning variables in the regression analysis of the stated EIs from Part A, and test for LIB.³⁶ A coefficient t-student test and a test of exclusion of each of these three variables test LIB. If none of b_2^h , b_3^h , and b_4^h , is significantly different from zero, then LIB does not prevent respondents from effectively performing the evaluation task of Part A. Table 1 shows that only two out of 21 dummy variables related to testing LIB are significant, but with small coefficients. Only one exclusion test is rejected ($P < 0.01$) – for the household type with 2 adults and 1 child. These findings offer supporting evidence that respondents’ own household type and/or level of material comfort do not bias their assessments of EIs in Part A.

Personal _Characteristics_i: testing whether personal characteristics of respondents affect their answers. This is a set of conditioning variables referring to personal characteristics of the respondents. A coefficient t-student test and a test of exclusion of each of these variables indicate whether any characteristics of the respondents affect their assessments of EI. With two exceptions, Table 2 shows that respondents’ personal characteristics do not appear statistically significant in the regressions. Respondents living in the New Laender report slightly higher ESs.³⁷ More educated respondents also state slightly higher ESs for hypothetical household types with children. Probably, more educated parents pursue higher education for their children. All significant coefficients are small.

Explanatory power of the regressions. The regressions fit the data quite well; they explain 30-54% of the total variation of stated ESs. Small standard errors for coefficients a^h and b^h in equation (20) indicate a broad consensus across respondents concerning the evaluation

³⁶Table A4 in Appendix A shows the household-type distribution of respondents who are included in the $LIB_{mc,i}$ dummy variable. This is a total of 415 respondents, the sum of the entries in the first column of Table A4. Each entry in the first column of Table A4 shows the number of respondents in the $(LIB_{h,i} \cdot LIB_{mc,i})$ dummy variable for each household type. Apart from single-adult households with two or three children, LIB tests based on the $(LIB_{h,i} \cdot LIB_{mc,i})$ dummy variable have sufficient statistical power.

³⁷This is consistent with the findings by Alesina and Fuchs-Schündeln (2007) regarding differences in opinions between East Germans and West Germans.

task of Part A.

Testing the affine relationship (18) using the six pilot surveys. As we discussed above, Figures 2 and 3 support the affine relationship given by (18) as well. Table 3 presents specification tests based on seemingly unrelated regressions (SUR) and the inclusion of dummies around the linear functional form having the same structure as in the regression model (19).³⁸ With the highest value of the F-test statistic being 1.75, in all 42 cases examined the affine relationship given by (18) is not rejected.

4. The Implied Research Agenda for Aggregation

Using our estimates from Table 1, and data taken from the German Income and Expenditure Survey in year 2003, the momentary utility function of the German representative consumer in 2003 is given by,

$$u^{RC}(c, t = 2003) = \frac{\left(c + \frac{14.91}{\alpha} \cdot \beta_{OMH} - EUR\ 3,281\right)^{1-\frac{1}{\alpha}} - 1}{1 - \frac{1}{\alpha}}, \quad (21)$$

where α is a free calibrating parameter, $-\beta_{OMH}/\alpha$ is the subsistence consumption of a one-member household, and the amount is in year 2003 Euros.³⁹ That all intercepts in Figure 1 are strictly positive advocates the existence of subsistence consumption, so some vector of parameters $\theta = (\alpha, \beta_{OMH})$ with $\alpha, \beta_{OMH} > 0$, seems more plausible for fitting micro data.

Using our survey's estimates as true without further verification in search of a reduced best-fitted aggregative model (with only a reduced set of parameters $\theta^* = (\alpha^*, \beta_{OMH}^*)$), would be narrow-minded and overly forward. The implied research agenda's success should

³⁸Since for each family type the same respondent has provided five equivalent income evaluations, the error terms across the seven family types might be cross correlated. This can generate a loss in the efficiency of estimators and can weaken the confidence in our specification tests. To cope with this problem we estimate a system of 7 seemingly unrelated regressions.

³⁹See Appendix D for the derivation of (21). In an aggregative model that uses the utility function given by (21), the appropriate measure of aggregate income to use is one-member-household equivalent income. In Appendix D we present how we construct the distribution of one-member-household equivalent incomes using data from the German Income and Expenditure Survey in year 2003.

rely upon best-fitting micro data, an extension we do not pursue in this paper. However, we have made an essential contribution towards the goals of examining *all possible best-fitted models with time-separable preference that imply aggregation*. This is because Theorem 2 gives the comprehensive class of time-separable utility functions that can serve as the key ingredient even in models with borrowing constraints and approximate aggregation properties. Theorem 2 can help in performing an exhaustive search among linear-aggregation models, subject to specification of micro-data processes that describe micro data in, say, heterogeneous-agent models in the fashion of Aiyagari (1994) or Krusell and Smith (1998), or in econometrically estimated demand systems in the fashion of Lewbel (1989) or Donaldson and Pendakur (2006).

Apart from contributing a guide concerning which class of models to investigate first, we also contribute an additional data set for as a complementary goodness-of-fit criterion to existing data: survey evaluations of EIs. Given that exact linear aggregation can be consistent with heterogeneous rates of time preference and elasticities of intertemporal substitution, we are confident that there is an exciting research agenda ahead. Of course, studying market incompleteness, borrowing constraints, and other nonconvexities may lead us to studying approximate aggregation. Nevertheless, with preferences from the class shown in Theorem 2, approximate aggregation properties are difficult to go away.⁴⁰

5. Conclusion

Aggregation is of central importance to micro-founded macro models that rely on estimation based on macro aggregates alone. At the same time, household-size heterogeneity is of key importance to the aggregation question, as household size influences consumer choices and

⁴⁰We thank Dirk Krueger for explaining this point to us in his formal discussion of an earlier version of this paper.

this influence is further channeled to both aggregate and household-level micro data. Here we provided aggregation results that pertain to a dynamic representative consumer with savings options and dynasties with changing family size over time.

The aggregation results in a dynamic context suggest that the aggregation-testing agenda is still open to further research. In particular, empirical analyses framed through static models which reject the Gorman requirement of linear and parallel Engel curves may suffer from imprecision due to missing variables. If households are forward looking, expectations of future events, e.g. changes in household income or composition, impact today's consumer choices. If these expectations are not adequately modeled, empirical tests have limited power.

It is crucial to study how household-size heterogeneity changes aggregation requirements. This is so because household-size heterogeneity and anticipation of future demographic changes of a household bring substantial preference heterogeneity to observed choices. We have arrived at a stringent and potentially fragile implication of household-size heterogeneity. We have shown that falsifying aggregation collapses into empirically rejecting a single equation regarding how much individuals benefit from within-household sharing of goods: once expenditures for subsistence needs are subtracted from permanent income, the remainder permanent incomes must entail the same household-size economies for the rich and for the poor. We have tested the validity of this equation without a-priori assumptions or conditioning on a model. We have not performed tests of linear consumption decision rules. Our tests have focused on the additional aggregation requirements introduced by household-size heterogeneity.

We have produced survey data from a large sample of respondents in Germany, both testing the critical equation for falsifying the representative consumer concept, and also

demonstrating the effectiveness of our survey instrument. In seven tests the representative consumer has not been falsified, at least not with high confidence in marginal cases. In 42 more tests using pilot data from six countries the representative consumer construct has never been falsified. Although our results do not prove the existence of a representative consumer, our findings offer external empirical validation that household-size heterogeneity may not hinder consumer-demand aggregation properties.

An extension is to examine aggregation conditions using non-time-separable utility.⁴¹ Improving survey instruments in future work should uncover more about the structure of utility that underlies observed consumer choices. It may also prove fruitful to combine survey data with observed choices. Survey data can add another goodness-of-fit criterion for estimating preference parameters, especially for multimember households, irrespective of whether there is within-household bargaining or not.

⁴¹For example, it may be that utility implying habit persistence, in some functional forms similar to these proposed by Constantinides (1990) might be both sufficient and necessary for aggregation, and it is interesting to examine whether equation (18) is also an implication of aggregation. This was beyond the scope of the present analysis which focused on time-separable preferences.

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Appendix – Proofs

Proof of Theorem 1

Part 1: Necessity

Fix any function $\rho : \mathbb{R}_+ \rightarrow \mathbb{R}_{++}$, and any collection $(u^i)_{i \in \mathcal{I}}$, with properties complying with Assumptions 1, 2, and 4. Assume that a representative consumer exists with some momentary utility function $v^{RC} : \mathbb{C}^{RC} \times \mathbb{R}_+ \rightarrow \mathbb{R}$, of the form $v^{RC}(c(t), t)$, at each point in time. Under Assumption 3, from Definition 1 and (5) it must be that,

$$\frac{v_1^{RC}(\int_{\mathcal{I}} c^i(t) d\mu(i), t)}{v_{11}^{RC}(\int_{\mathcal{I}} c^i(t) d\mu(i), t)} \left[r(t) + \frac{v_{12}^{RC}(\int_{\mathcal{I}} c^i(t) d\mu(i), t)}{v_1^{RC}(\int_{\mathcal{I}} c^i(t) d\mu(i), t)} \right] = \int_{\mathcal{I}} \mu(i) \frac{u_1^i(c^i(t))}{u_{11}^i(c^i(t))} di [r(t) - \rho(t)] , \quad (22)$$

where the term

$$-\frac{v_{12}^{RC}(\int_{\mathcal{I}} c^i(t) d\mu(i), t)}{v_1^{RC}(\int_{\mathcal{I}} c^i(t) d\mu(i), t)}$$

is the temporal rate of time preference of the representative consumer.

(Necessity) Step 1: preliminary characterization of the function $\int_0^\infty v^{RC}(c(t), t) dt$.

According to Definition 1, the existence (and the implied preference primitives) of the representative consumer should be independent from any price regime. The case where $r(t) = \rho(t)$ for all $t \geq 0$, should always be included in the price domain. To see this, fix any moment in time, $t \in \mathbb{R}_+$, pick any household $i \in \mathcal{I}$, and multiply her budget constraint, (3), by the integrating factor $e^{-\int_t^\tau r(s) ds}$, integrate over all $\tau \in [t, \infty)$, and apply the transversality condition, to obtain,

$$\int_t^\infty e^{-\int_t^\tau r(s) ds} c^i(\tau) d\tau = a^i(t) + \int_t^\infty e^{-\int_t^\tau r(s) ds} \theta^i(\tau) w(\tau) d\tau . \quad (23)$$

For the case $r(t) = \rho(t)$ for all $t \geq 0$, under Assumption 3, (5) implies that $\dot{c}^i(t) = 0$ for all $t \in \mathbb{R}_+$, and all $i \in \mathcal{I}$, so, (23) implies that

$$c^i(t) = \hat{c}^i = \frac{a^i(t) + \int_t^\infty e^{-\int_t^\tau \rho(s) ds} \theta^i(\tau) w(\tau) d\tau}{\int_t^\infty e^{-\int_t^\tau \rho(s) ds} d\tau}, \text{ for all } t \geq 0. \quad (24)$$

For the given preference profile, $(u^i)_{i \in \mathcal{I}}$, (24) implies that there are always $(a_0^i, \theta^i)_{i \in \mathcal{I}}$ and $(w(t))_{t \geq 0}$ securing that $\hat{c}^i \in \mathbb{C}^i$ for all $i \in \mathcal{I}$, and for all $t \geq 0$. So, the case $r(t) = \rho(t)$ for all $t \geq 0$, is always part of the domain complying with Assumption 3, for any $(u^i)_{i \in \mathcal{I}}$ that satisfies Assumptions 1, 2, and 4.

Thus, set $r(t) = \rho(t)$ for all $t \geq 0$ and pick an appropriate $(a_0^i, \theta^i)_{i \in \mathcal{I}}$ and $(w(t))_{t \geq 0}$ securing that $\hat{c}^i > \underline{c}^i$ for all $i \in \mathcal{I}$, and for all $t \geq 0$, and also set,

$$c \equiv \int_{\mathcal{I}} \hat{c}^i d\mu(i) .$$

Equations (22) and (24) imply that the necessary optimality conditions of the representative consumer are,

$$-\frac{v_{12}^{RC}(c, t)}{v_1^{RC}(c, t)} = \rho(t) .$$

So, standard Riemann integration with respect to t over the time interval $[0, t]$ implies that,

$$v_1^{RC}(c, t) = e^{-\int_0^t \rho(\tau) d\tau} v_1^{RC}(c, 0) ,$$

or,

$$v^{RC}(c, t) = e^{-\int_0^t \rho(\tau) d\tau} v^{RC}(c, 0) ,$$

ignoring the constant, since this is a utility function. Setting,

$$u^{RC}(c) \equiv v^{RC}(c, 0) ,$$

we conclude that the objective of the representative consumer must be of the form,

$$U^{RC}((c(t))_{t \geq 0}) = \int_0^\infty e^{-\int_0^t \rho(\tau) d\tau} u^{RC}(c(t)) dt . \quad (25)$$

For notational ease, let $f^{RC} : \mathbb{C}^{RC} \rightarrow \mathbb{R}_{++}$ and $(f^i : \mathbb{C}^i \rightarrow \mathbb{R}_{++})_{i \in \mathcal{I}}$, with

$$f^{RC}(\cdot) = -\frac{v_1^{RC}(\cdot)}{v_{11}^{RC}(\cdot)} \quad \text{and} \quad f^i(\cdot) = -\frac{u_1^i(\cdot)}{u_{11}^i(\cdot)} \quad \text{for all } i \in \mathcal{I}.$$

Combining (25) with (22), it is,

$$f^{RC} \left(\int_{\mathcal{I}} c^i(t) d\mu(i) \right) = \int_{\mathcal{I}} f^i(c^i(t)) d\mu(i) , \quad (26)$$

for all $(c^i(t) \in \mathbb{C}^i)_{i \in \mathcal{I}}$ that are consumer-equilibrium choices and $t \geq 0$.

(Necessity) Step 2: characterization of $f^{RC} : \mathbb{R}_+ \rightarrow \mathbb{R}_{++}$ and $(f^i : \mathbb{R}_+ \rightarrow \mathbb{R}_{++})_{i \in \mathcal{I}}$. In this step we show that,

$$(26) \Leftrightarrow \left\{ \begin{array}{l} f^i(c) = \alpha c + \beta_i, \quad \text{and,} \\ f^{RC}(c) = \alpha c + \int_{\mathcal{I}} \beta_i d\mu(i) , \\ \text{for some } \alpha \in \mathbb{R} \text{ and some } \beta_i \in \mathbb{R}, \text{ for all } i \in \mathcal{I} \end{array} \right\} . \quad (27)$$

The sufficiency part of (27) is straightforward. For the necessity part of (27), let (26) hold, being the only information available concerning $f^{RC} : \mathbb{R}_+ \rightarrow \mathbb{R}_{++}$ and the collection $(f^i : \mathbb{R}_+ \rightarrow \mathbb{R}_{++})_{i \in \mathcal{I}}$. Suppose that $r(t) = \rho(t)$ for all $t \geq 0$, and, given (24), find a common distribution of $(a_0^i, \theta^i)_{i \in \mathcal{I}}$ and $(w(t))_{t \geq 0}$, where $a_0^i = a_0$ and $\theta^i = \theta$, so that $c^i(t) = \tilde{c}$ for all $i \in \mathcal{I}$, and all $t \geq 0$, also with $\tilde{c} \in \bigcap_{i \in \mathcal{I}} \mathbb{C}^i$.

Let,

$$\Phi^{RC}(c) \equiv f^{RC}(c) - f^{RC}(\tilde{c}) , \quad (28)$$

and,

$$\Phi^i(c) \equiv f^i(c) - f^i(\tilde{c}) \text{ , for all } i \in \mathcal{I} \text{ .} \quad (29)$$

For this distribution, (26) implies that,

$$f^{RC}(\tilde{c}) = \int_{\mathcal{I}} f^i(\tilde{c}) d\mu(i) \text{ .} \quad (30)$$

Given (1), set $\underline{\mu}$ such that,

$$0 < \underline{\mu} \leq \inf \{d\mu(i) \mid i \in \mathcal{I}\} \text{ .} \quad (31)$$

Pick any arbitrary household type $i \in \mathcal{I}$, keep prices as before, and modify the previous distribution by adding to $\underline{\mu}$ of this household type different wealth or productivity that yields $c^i(t) = (\tilde{c} + \Delta c) \in \bigcap_{i \in \mathcal{I}} \mathbb{C}^i$, for all $t \geq 0$. Since prices are the same, $c^j(t) = \tilde{c}$, for all $j \in \mathcal{I} \setminus \{i\}$ and for some households of type i with density $d\mu(i) - \underline{\mu}$, and for all $t \geq 0$. Combining (26), (30), (28) and (29), it is,

$$\Phi^{RC}(\underline{\mu}\Delta c + \tilde{c}) = \underline{\mu}\Phi^i(\Delta c + \tilde{c}) \text{ .} \quad (32)$$

Since the choices of $i \in \mathcal{I}$, Δc , and $\tilde{c} \in \bigcap_{i \in \mathcal{I}} \mathbb{C}^i$, were arbitrary, and since we can construct the same distribution of consumption choices for all $i \in \mathcal{I}$, (32) holds for all $i \in \mathcal{I}$, so,

$$\Phi^i(c) = \Phi(c) \text{ for all } c \in \bigcap_{i \in \mathcal{I}} \mathbb{C}^i \text{ and for all } i \in \mathcal{I} \text{ .} \quad (33)$$

Given (24), we are able to construct any interior optimal path with distribution of consumptions with $c^i(t) = c \in \bigcap_{i \in \mathcal{I}} \mathbb{C}^i$ for all $i \in \mathcal{I}$, and all $t \geq 0$. Therefore, (26), (30), and (33) imply that,

$$\Phi^{RC}(c) = \Phi^i(c) = \Phi(c) \text{ for all } c \in \bigcap_{i \in \mathcal{I}} \mathbb{C}^i \text{ and for all } i \in \mathcal{I} \text{ ,} \quad (34)$$

and,

$$\Phi\left(\int_{\mathcal{I}} c^i(t) d\mu(i)\right) = \int_{\mathcal{I}} \Phi(c^i(t)) d\mu(i) \text{ , for all } \left(c^i(t) \in \bigcap_{i \in \mathcal{I}} \mathbb{C}^i\right)_{i \in \mathcal{I}} \text{ , and } t \geq 0 \text{ ,} \quad (35)$$

holding for the whole domain of wealth/labor-productivity heterogeneity and prices where household choices fall in the interval $\bigcap_{i \in \mathcal{I}} \mathbb{C}^i$ (see Assumption 4) and are interior. Equation (35) enables us to further characterize Φ . In particular,

$$(35) \Leftrightarrow \Phi \text{ is affine on } \bigcap_{i \in \mathcal{I}} \mathbb{C}^i. \quad (36)$$

The sufficiency part of (36) is straightforward, so for the necessity part of (36) let's set,

$$z^i \equiv c^i - \tilde{c}, \quad (37)$$

with \tilde{c} defined as above for an arbitrary $\tilde{c} \in \bigcap_{i \in \mathcal{I}} \mathbb{C}^i$, in the case where $r(t) = \rho(t)$ for all $t \geq 0$. So, fix \tilde{c} and set,

$$\Psi(z) \equiv \Phi(z) - \Phi(0), \quad (38)$$

since we know that for the transformed variable, z , the choice of 0 falls in the class of interior solutions to a distribution in the domain of $(u^i)_{i \in \mathcal{I}}$, namely the case where all households choose $\tilde{c} \in \bigcap_{i \in \mathcal{I}} \mathbb{C}^i$ at all times. We now show that Ψ is a linear functional. For any partition of households, irrespective of their household types, say, $\mathcal{I}_1, \mathcal{I}_2 \subset \mathcal{I}$, with $\mathcal{I}_1 \cap \mathcal{I}_2 = \emptyset$, and $\int_{\mathcal{I}_1} d\mu(i) = \mu$, retaining $r(t) = \rho(t)$ for all $t \geq 0$, provide the same a_0 and a labor-productivity function θ to all $i \in \mathcal{I}_1$, so that consumption is equal to $(\Delta c + \tilde{c}) \in \bigcap_{i \in \mathcal{I}} \mathbb{C}^i$ for all $i \in \mathcal{I}_1$ at all times, provide to the remaining households \tilde{a}_0 and a labor-productivity $\tilde{\theta}$, so that their consumption is equal to $\tilde{c} \in \bigcap_{i \in \mathcal{I}} \mathbb{C}^i$ for all $i \in \mathcal{I}_2$ at all times. Then, $z^i = \Delta c$ for all $i \in \mathcal{I}_1$, and $z^i = 0$ for all $i \in \mathcal{I}_2$, so,

$$\Phi(\mu \Delta c) = \Phi(\mu \Delta c + (1 - \mu) 0),$$

and (35) and (38) imply that,

$$\Phi(\mu \Delta c) = \mu \Phi(\Delta c) + (1 - \mu) \Phi(0),$$

or,

$$\Psi(\mu\Delta c) = \mu\Psi(\Delta c) . \quad (39)$$

Notice that the choices of Δc and μ were arbitrary. So, we can take any $\mu_1, \mu_2 \in (0, 1)$ with $(\mu_1\Delta c + \tilde{c}), (\mu_2\Delta c + \tilde{c}) \in \bigcap_{i \in \mathcal{I}} \mathbb{C}^i$ and $\frac{\mu_2}{\mu_1} = \xi \in \mathbb{R}_+$. Repeating the same steps, (39) yields $\Psi(\mu_1\Delta c) = \mu_1\Psi(\Delta c)$ and $\Psi(\xi\mu_1\Delta c) = \xi\mu_1\Psi(\Delta c)$, or,

$$\Psi(\xi\mu_1\Delta c) = \xi\Psi(\mu_1\Delta c), \quad \text{for all } \xi \in \mathbb{R}_+ . \quad (40)$$

Since Ψ is a univariate function, (40) is sufficient to prove that Ψ is linear. So, let,

$$\Psi(z) = \alpha z, \quad \alpha \in \mathbb{R},$$

and, due to the linearity of Ψ , the transformation (37) can be ignored, having (38) and (34) implying that, $\Phi(c) = \alpha c + \Phi(0)$. But since (28) and (29) imply that $\Phi(\tilde{c}) = 0, \Phi(0) = -\alpha\tilde{c}$. So,

$$\Phi^{RC}(c) = \Phi^i(c) = \Phi(c) = \alpha c - \alpha\tilde{c}, \quad \alpha \in \mathbb{R}, \text{ for all } c \in \bigcap_{i \in \mathcal{I}} \mathbb{C}^i \text{ and for all } i \in \mathcal{I} . \quad (41)$$

Using (41) we show that,

$$\Phi^i(c) = \Phi(c) = \alpha c - \alpha\tilde{c}, \quad \alpha \in \mathbb{R}, \text{ for all } c \in \mathbb{C}^i \text{ and for all } i \in \mathcal{I} . \quad (42)$$

To prove (42), consider the case where an arbitrary $c^j \in \mathbb{C}^j$ is such that $c^j \leq \inf \left(\bigcap_{i \in \mathcal{I}} \mathbb{C}^i \right)$ or $c^j \geq \sup \left(\bigcap_{i \in \mathcal{I}} \mathbb{C}^i \right)$ for some $j \in \mathcal{I}$, whenever any of the two is possible (i.e. whenever $\inf \left(\bigcap_{i \in \mathcal{I}} \mathbb{C}^i \right) > 0$, or $\sup \left(\bigcap_{i \in \mathcal{I}} \mathbb{C}^i \right) < \infty$). It is always that there exists some $\mu \in (0, 1)$, with $\mu \leq d\mu(j)$, such that $(\mu c^j + (1 - \mu)\tilde{c}) \in \bigcap_{i \in \mathcal{I}} \mathbb{C}^i$. So, retaining $r(t) = \rho(t)$ for all $t \geq 0$, provide a level a_0 and a labor-productivity function θ to a mass μ of type $j \in \mathcal{I}$, so that consumption is equal to c^j at all times, and also provide to the remaining households \tilde{a}_0

and a labor-productivity $\tilde{\theta}$, so that their consumption is equal to $\tilde{c} \in \bigcap_{i \in \mathcal{I}} \mathbb{C}^i$ at all times.

Combining (26), (28), (29) and (30), it is,

$$\mu \Phi^j (c^j) = \Phi^{RC} (\mu c^j + (1 - \mu) \tilde{c}) .$$

But since $(\mu c^j + (1 - \mu) \tilde{c}) \in \bigcap_{i \in \mathcal{I}} \mathbb{C}^i$, (41) implies that $\Phi^{RC} (\mu c^j + (1 - \mu) \tilde{c}) = \alpha (\mu c^j + (1 - \mu) \tilde{c}) - \alpha \tilde{c}$, or

$$\Phi^j (c^j) = \alpha c^j - \alpha \tilde{c} .$$

Since the choices of $j \in \mathcal{I}$ and $c^j \in \mathbb{C}^j$ were arbitrary, (42) is proved.

Combining (29) with (42) it is,

$$f^i (c) = \alpha c - \alpha \tilde{c} + f^i (\tilde{c}) \quad \text{for all } c \in \mathbb{C}^i \text{ and all } i \in \mathcal{I} . \quad (43)$$

Now that all f^i 's are completely characterized over their domains, \mathbb{C}^i , we can consider the case of $c = 0$, irrespective from whether $0 \in \mathbb{C}^i$ or not, in order to set the intercepts of all f^i 's. Equation (43) implies,

$$f^i (\tilde{c}) = \alpha \tilde{c} + f^i (0) . \quad (44)$$

Setting $f^i (0) = \beta_i$ for some $\beta_i \in \mathbb{R}$, for all $i \in \mathcal{I}$, a final combination of (43) with (44), and also setting $\beta_{RC} = \int_{\mathcal{I}} \beta_i d\mu (i)$ (consistently with (26)), completes the proof of (27).

(Necessity) Step 3: characterization of $(u^i : \mathbb{R}_+ \rightarrow \mathbb{R})_{i \in \mathcal{I}}$ and $u^{RC} : \mathbb{R}_+ \rightarrow \mathbb{R}$.

In light of (27), we derive the functional forms of utility for all household types through Riemann integration. There are two general cases, these of $\alpha \neq 0$ and $\alpha = 0$. (The case where $\alpha = 1$ is also of special interest, but the particular functional form of $(u^i)_{i \in \mathcal{I}}$ and u^{RC} that result in this case, can be derived from the more general functional forms that apply to $\alpha \neq 0$.)

For the case where $\alpha \neq 0$, (27) implies that,

$$\frac{u_{11}^i(c)}{u_1^i(c)} = -\frac{1}{\alpha c + \beta_i}, \quad (45)$$

and the indefinite Riemann integral of this expression with respect to c yields,

$$\ln [u_1^i(c)] = -\frac{1}{\alpha} \ln(\alpha c + \beta_i) + \kappa_i, \quad (46)$$

where κ_i is some constant in \mathbb{R} , that can be household-specific, and integrating once more, it is,

$$u^i(c) = e^{\kappa_i} \frac{(\alpha c + \beta_i)^{1-\frac{1}{\alpha}}}{\alpha(1-\frac{1}{\alpha})} + \kappa, \quad (47)$$

where κ is, again some constant. Setting $e^{\kappa_i} = 1$, without loss of generality, and κ accordingly, we obtain the result of (8). The special case where $\alpha = 1$, is known to yield the result that $u^i(c) = \ln(\alpha c + \beta_i) + \kappa$, through computing the limit of the above expression for $\alpha \rightarrow 1$ using L'Hôpital's rule. The preferences of the representative consumer are derived in the same way.

For the case where $\alpha = 0$,

$$\frac{u_{11}^i(c)}{u_1^i(c)} = -\frac{1}{\beta_i}, \quad (48)$$

and in order for $u_1^i > 0$ and $u_{11}^i < 0$ to hold, it must be that $\beta_i > 0$. So,

$$\ln [u_1^i(c)] = -\frac{1}{\beta_i} c + \kappa_i, \quad (49)$$

and,

$$u^i(c) = -\frac{e^{\kappa_i}}{\beta_i} e^{-\frac{1}{\beta_i} c} + \kappa, \quad (50)$$

so, setting $\frac{e^{\kappa_i}}{\beta_i} = 1$ and $\kappa = 0$ yields the corresponding function in (8). With the same reasoning for the representative consumer, the proof of the necessity part is complete.

Part 2: Sufficiency

The particular functional forms given by (8) enable a complete analytical characterization of the demand functions of all households at all times. Again, two cases must be examined separately, this of $\alpha \neq 0$ and the case where $\alpha = 0$.

Under the assumption that $\alpha \neq 0$, (5), implies,

$$\dot{c}^i(t) = [\alpha c^i(t) + \beta_i] [r(t) - \rho(t)] ,$$

so, multiplying this expression by the integrating factor $e^{-\alpha \int_t^\tau [r(s) - \rho(s)] ds}$ and integrating over the interval $[t, \tau]$ for any $\tau \in [t, \infty)$, yields,

$$c^i(\tau) = c^i(t) e^{\alpha \int_t^\tau [r(s) - \rho(s)] ds} + \beta_i e^{\alpha \int_t^\tau [r(s) - \rho(s)] ds} \int_t^\tau e^{-\alpha \int_t^\tau [r(s) - \rho(s)] ds} [r(s) - \rho(s)] ds .$$

Multiplying this last expression by $e^{-\int_t^\tau r(s) ds}$, integrating over all $\tau \in [t, \infty)$, and combining the result with (23), gives,

$$c^i(t) = \frac{a^i(t) + \int_t^\infty e^{-\int_t^\tau r(s) ds} \theta^i(\tau) w(\tau) d\tau}{\int_t^\infty e^{\int_t^\tau [(\alpha-1)r(s) - \alpha\rho(s)] ds} d\tau} - \frac{\beta_i \int_t^\infty e^{\int_t^\tau [(\alpha-1)r(s) - \alpha\rho(s)] ds} \int_t^\tau e^{-\alpha \int_t^\tau [r(s) - \rho(s)] ds} [r(s) - \rho(s)] ds d\tau}{\int_t^\infty e^{\int_t^\tau [(\alpha-1)r(s) - \alpha\rho(s)] ds} d\tau} , \quad (51)$$

which can be linearly aggregated across all a^i 's, θ^i 's and β_i 's, proving that a representative consumer exists, as long as Assumption 1 holds, which keeps all individual demands taking the form of (51).

For the case where $\alpha = 0$, when all individual utilities fall in the class of $u^i(c) = -e^{-\frac{1}{\beta_i}c}$, (51) implies that,

$$c^i(t) = \frac{a^i(t) + \int_t^\infty e^{-\int_t^\tau r(s) ds} \theta^i(\tau) w(\tau) d\tau - \beta_i \int_t^\infty e^{-\int_t^\tau r(s) ds} \int_t^\tau [r(s) - \rho(s)] ds d\tau}{\int_t^\infty e^{-\int_t^\tau r(s) ds} d\tau} , \quad (52)$$

which can also be linearly aggregated across all a^i 's, θ^i 's and β_i 's, completing the proof of the theorem. *Q.E.D.*

Proof of Theorem 2

Part 1: Necessity

Fix any function $\rho : \mathbb{R}_+ \rightarrow \mathbb{R}_{++}$, and any collection $(u^i)_{i \in \mathcal{I}}$, with properties complying with Assumptions 2, 3, and 5 through 7. Assume that a representative consumer exists with some momentary utility function $v^{RC} : \mathbb{C}^{RC} \times \mathbb{R}_+ \rightarrow \mathbb{R}$, of the form $v^{RC}(c(t), t)$, at each point in time.

Considering any $i \in \mathcal{I}$, its optimality conditions imply that,

$$-\frac{u_{11}^i(c^i(t), t)}{u_1^i(c^i(t), t)} \dot{c}^i(t) - \frac{u_{12}^i(c^i(t), t)}{u_1^i(c^i(t), t)} = r(t) - \rho(t) \ , \quad t \geq 0 \ . \quad (53)$$

Now pick $r(t) = \rho(t)$ for all $t \geq 0$, substitute it to (53) and take the indefinite integral with respect to time to get,

$$u_1^i(c^i(t), t) = \kappa \ , \quad t \geq 0 \ . \quad (54)$$

where κ is some constant. Due to the fact that $u_{11}^i(c^i(t), t) < 0$, and due to Assumptions 6 and 7, there is always a $\kappa > 0$ such that $c^i(t) \in \mathbb{C}^i(t)$ for all $t \geq 0$, satisfying (54). For $r(t) = \rho(t)$, (53) implies that,

$$\dot{c}^i(t) = -\frac{u_{12}^i(c^i(t), t)}{u_{11}^i(c^i(t), t)} \ . \quad (55)$$

The level of κ in (54) will be uniquely identified by setting $u_1^i(c^i(0), 0) = \kappa$ and applying (23) at time 0, combined with the dynamics of $c^i(t)$ implied by (55). Due to Assumption 6, such an interior path exists on \mathfrak{C}^i , as \mathfrak{C}^i is defined in Assumption 7. This means that with the right choices of initial wealth and labor productivity, we can construct interior paths that span \mathfrak{C}^i . Moreover, always for the case where $r(t) = \rho(t)$ for all $t \geq 0$, due to Assumption 7, for any $i \in \mathcal{I}$, we can generate any choice of $c \in \bigcap_{i \in \mathcal{I}} \mathfrak{C}^i$ at any point in time, picking the appropriate initial wealth and labor productivity, since the dynamics of consumption are solely driven by (55).

With this facility at hand, we can look at the problem of the representative consumer, whose optimal Euler equation gives,

$$-\frac{v_{11}^{RC} \left(\int_{\mathcal{I}} c^i(t) d\mu(i), t \right)}{v_1^{RC} \left(\int_{\mathcal{I}} c^i(t) d\mu(i), t \right)} \int_{\mathcal{I}} \dot{c}^i(t) d\mu(i) - \frac{v_{12}^{RC} \left(\int_{\mathcal{I}} c^i(t) d\mu(i), t \right)}{v_1^{RC} \left(\int_{\mathcal{I}} c^i(t) d\mu(i), t \right)} = r(t) \quad , \quad t \geq 0 \quad , \quad (56)$$

and combining it with (53), it is,

$$\begin{aligned} & \frac{v_{11}^{RC} \left(\int_{\mathcal{I}} c^i(t) d\mu(i), t \right)}{v_{11}^{RC} \left(\int_{\mathcal{I}} c^i(t) d\mu(i), t \right)} r(t) + \frac{v_{12}^{RC} \left(\int_{\mathcal{I}} c^i(t) d\mu(i), t \right)}{v_1^{RC} \left(\int_{\mathcal{I}} c^i(t) d\mu(i), t \right)} = \\ & = [r(t) - \rho(t)] \int_{\mathcal{I}} \frac{u_1^i(c^i(t), t)}{u_{11}^i(c^i(t), t)} d\mu(i) + \int_{\mathcal{I}} \frac{u_{12}^i(c^i(t), t)}{u_{11}^i(c^i(t), t)} d\mu(i) \quad . \end{aligned} \quad (57)$$

Setting $r(t) = \rho(t)$ for all $t \geq 0$, (57) becomes,

$$\frac{v_{11}^{RC} \left(\int_{\mathcal{I}} c^i(t) d\mu(i), t \right)}{v_{11}^{RC} \left(\int_{\mathcal{I}} c^i(t) d\mu(i), t \right)} \rho(t) + \frac{v_{12}^{RC} \left(\int_{\mathcal{I}} c^i(t) d\mu(i), t \right)}{v_1^{RC} \left(\int_{\mathcal{I}} c^i(t) d\mu(i), t \right)} = \int_{i \in \mathcal{I}} \frac{u_{12}^i(c^i(t), t)}{u_{11}^i(c^i(t), t)} d\mu(i) \quad . \quad (58)$$

But since, as explained above, for the case where $r(t) = \rho(t)$ for all $t \geq 0$, one can generate any distribution of consumption choices, (58) holds for the whole domain implied by Assumption 3. So, substituting (58) into (57), it is,

$$\frac{v_{11}^{RC} \left(\int_{\mathcal{I}} c^i(t) d\mu(i), t \right)}{v_{11}^{RC} \left(\int_{\mathcal{I}} c^i(t) d\mu(i), t \right)} = \int_{i \in \mathcal{I}} \frac{u_1^i(c^i(t), t)}{u_{11}^i(c^i(t), t)} d\mu(i) \quad , \quad (59)$$

for the whole domain implied by Assumption 3, including the case where $r(t) = \rho(t)$ for all $t \geq 0$. But then, for any $t \geq 0$, the same argument that was developed in step 2 of the necessity part of the proof of Theorem 1, to get,

$$\begin{aligned} & \frac{u_1^i(c, t)}{u_{11}^i(c, t)} = \alpha(t) c + \beta^i(t) \quad , \quad \text{and,} \\ & \frac{v_{11}^{RC}(c, t)}{v_1^{RC}(c, t)} = \alpha(t) c + \int_{\mathcal{I}} \beta^i(t) d\mu(i) \quad , \end{aligned} \quad (60)$$

for some $\alpha(t) \in \mathbb{R}$ and some $\beta^i(t) \in \mathbb{R}$, for all $i \in \mathcal{I}$, $t \geq 0$

Using (60), with the same procedure as in step 3 of the necessity part of Theorem 1, candidate utility functions arise. Deriving individual demands, one can verify that this is possible only if

$$\alpha(t) = \alpha \neq 0 \quad , \quad \text{and } \beta^i(t) \text{ meeting Assumptions 6, 7, } t \geq 0 \quad ,$$

and

$$\alpha = 0, \quad \beta^i(t) = \beta_i G(t) ,$$

that match the utility functions of the theorem. In particular, for the case where $\alpha \neq 0$, demands are,

$$c^i(t) = \frac{a^i(t) + \int_t^\infty e^{-\int_t^\tau r(s)ds} \theta^i(\tau) w(\tau) d\tau + \frac{1}{\alpha} \int_t^\infty e^{\int_t^\tau [(\alpha-1)r(s) - \alpha\rho(s)]ds} \beta^i(\tau) d\tau}{\int_t^\infty e^{\int_t^\tau [(\alpha-1)r(s) - \alpha\rho(s)]ds} d\tau} - \frac{\beta^i(t)}{\alpha} , \quad (61)$$

which are linear with respect to β^i 's. On the contrary, the demands for the utility function,

$$u^i(c, t) = -e^{-\frac{1}{\beta^i(t)}c} ,$$

are,

$$c^i(t) = \frac{a^i(t) + \int_t^\infty e^{-\int_t^\tau r(s)ds} \theta^i(\tau) w(\tau) d\tau - \int_t^\infty e^{-\int_t^\tau r(s)ds} \beta^i(\tau) \int_t^\tau [r(s) - \rho(s)] ds d\tau}{\int_t^\infty e^{-\int_t^\tau r(s)ds} \frac{\beta^i(\tau)}{\beta^i(t)} d\tau} , \quad (62)$$

which can be linearly aggregated only if $\frac{\beta^i(t)}{\beta^i(0)} = \frac{\beta^j(t)}{\beta^j(0)}$ for all $i, j \in \mathcal{I}$, i.e. only when $\beta^i(t) = \beta_i G(t)$, $\beta_i > 0$ for all $i \in \mathcal{I}$, completing the necessity part.

Part 2: Sufficiency

Follows by (61) and (62), observing that, under the statement of the theorem, they are linear with respect to a^i 's, θ^i 's and β^i 's. *Q.E.D.*

Proof of Proposition 1

Suppose that a representative consumer exists, which gives rise to a preference profile $(u^i)_{i \in \mathcal{I}}$ characterized by (27) that appears in the proof of Theorem 1.

In the case where $\alpha = 0$, equation (27) implies equation (48) which can be integrated with respect to c to yield equations (49) and (50). Under all assumptions made in the statement of Proposition 1, equation (17) holds for all $i, j \in \mathcal{I}$, but for the empirical identification of any pair $(\bar{y}_E^i, \bar{y}_E^j)$, $i, j \in \mathcal{I}$ (for avoiding indeterminacy), constants added to the functional form (50) cannot be different across any two $i, j \in \mathcal{I}$. On the contrary, parameter κ_i in (50) can differ across $i, j \in \mathcal{I}$, since κ_i determines the level of i 's marginal utility of consumption in relation to the marginal utility of all other household types. So, combining equations (17) and (49) implies

$$\bar{y}_E^j = \beta_j (\kappa_j - \kappa_i) + \frac{\beta_j}{\beta_i} \bar{y}_E^i ,$$

which is consistent with (18).

In the case where $\alpha \neq 0$, and $\alpha \neq 1$, from the proof of Theorem 1 we can see that equation (47) holds, under the constraint that constants added to the functional form (47) cannot be different across any two $i, j \in \mathcal{I}$, for enabling the empirical identification of any pair $(\bar{y}_E^i, \bar{y}_E^j)$, $i, j \in \mathcal{I}$, while parameter κ_i can differ across $i, j \in \mathcal{I}$, $i \neq j$. Equations (17) and (47) imply,

$$\bar{y}_E^j = \frac{e^{\frac{\alpha}{\alpha-1}(\kappa_i - \kappa_j)} \beta_i - \beta_j}{\alpha} + e^{\frac{\alpha}{\alpha-1}(\kappa_i - \kappa_j)} \bar{y}_E^i , \quad (63)$$

which is also consistent with (18).

For the case where $\alpha = 1$, the existence of a representative consumer and empirical identification of any pair $(\bar{y}_E^i, \bar{y}_E^j)$, $i, j \in \mathcal{I}$, imply that $\kappa_i = \kappa_j = 0$ for all $i, j \in \mathcal{I}$. To see this, suppose, that, to the contrary, for some $i, j \in \mathcal{I}$, $i \neq j$, it is $\kappa_i \neq \kappa_j$ and also $\kappa_i \neq 0$,

without loss of generality. Then equation (46) implies, after setting $\alpha = 1$, that

$$u_1^i(c) = \frac{e^{\kappa_i}}{c + \beta_i} . \quad (64)$$

Integrating (64) with respect to c yields,

$$u^i(c) = e^{\kappa_i} \ln(c + \beta_i) + b , \quad (65)$$

where b is some constant which is common across all household types. Alternatively, (64) can be re-written as

$$u_1^i(c) = \frac{1}{e^{-\kappa_i}c + e^{-\kappa_i}\beta_i} ,$$

and integrating this last equation with respect to c gives,

$$u^i(c) = e^{\kappa_i} [\ln(c + \beta_i) - \kappa_i] + b . \quad (66)$$

Comparing (65) with (66) implies that the constant b must be adjusted for $i \in \mathcal{I}$, which contradicts the requirement that any pair $(\bar{y}_E^i, \bar{y}_E^j)$, $i, j \in \mathcal{I}$, can be uniquely identified by data by not allowing household-type specific constants to be added to utility functions. So, the only way that (65) and (66) coincide is setting $\kappa_i = 0$. Due to the arbitrary choice of i , it is $\kappa_i = \kappa_j = 0$ for all $i, j \in \mathcal{I}$, and (17) implies

$$\bar{y}_E^j = \beta_i - \beta_j + \bar{y}_E^i , \quad (67)$$

which is the special case of (63) with $\kappa_i = \kappa_j = 0$ for all $i, j \in \mathcal{I}$ and with $\alpha = 1$. In fact, setting $\alpha = 1$ in (63) directly implies that it can only be $\kappa_i = \kappa_j = 0$ for all $i, j \in \mathcal{I}$. Since (67) is also consistent with (18), the proposition is proved. Q.E.D.

Table 1. Summary of ordinary least squares regressions. Endogenous variable: equivalence scales stated by respondents. Number of observations: 2,042. Standard Errors in parentheses. P-values of F-tests in brackets. *** P<0.001, ** P<0.01, * P<0.05.

	Household type						
	1 adult, 1 child	1 adult, 2 children	1 adult, 3 children	2 adults, 0 children	2 adults, 1 child	2 adults, 2 children	2 adults, 3 children
Constant	1.06*** (0.03)	1.12*** (0.05)	1.20*** (0.08)	1.42*** (0.06)	1.44*** (0.07)	1.53*** (0.09)	1.61*** (0.11)
Reciprocal of reference income	269.74*** (9.77)	498.34*** (16.28)	728.85*** (23.45)	329.38*** (15.91)	592.99*** (20.81)	839.25*** (27.41)	1,079.86*** (34.34)
Dummy reference income 1,250 Euros	0.00 (0.01)	-0.00 (0.02)	-0.02 (0.03)	0.03 (0.02)	0.00 (0.03)	-0.02 (0.03)	-0.04 (0.04)
Dummy reference income 2,000 Euros	0.02 (0.01)	0.02 (0.02)	0.02 (0.02)	0.00 (0.02)	-0.00 (0.02)	-0.00 (0.03)	-0.02 (0.03)
Dummy reference income 2,750 Euros	-0.02* (0.01)	-0.04** (0.02)	-0.07** (0.02)	-0.05* (0.02)	-0.08** (0.02)	-0.11*** (0.03)	-0.13*** (0.04)
Normalized Likert scale evaluation	-0.04*** (0.01)	-0.07*** (0.01)	-0.10*** (0.02)	-0.05*** (0.01)	-0.07*** (0.02)	-0.09*** (0.02)	-0.13*** (0.02)
Dummy for same household type of respondent	0.04 (0.02)	-0.01 (0.05)	-0.14* (0.06)	0.02 (0.03)	0.02 (0.03)	0.01 (0.04)	0.01 (0.08)
Dummy for same material comfort of respondent	-0.01 (0.01)	-0.03 (0.02)	-0.03 (0.03)	-0.04 (0.02)	-0.00 (0.03)	-0.03 (0.04)	-0.05 (0.04)
Dummy for same household type and material comfort of respondent	-0.06 (0.04)	0.13 (0.11)	-0.03 (0.11)	0.05 (0.05)	-0.16* (0.07)	-0.02 (0.08)	-0.04 (0.14)
Adjusted R ²	0.46	0.53	0.54	0.30	0.46	0.52	0.54
F test statistic for exclusion of all reference income dummy variables	2.36 [0.07]	3.07* [0.03]	3.29* [0.02]	3.60* [0.01]	3.37* [0.02]	3.45* [0.02]	3.51* [0.01]
F test statistic for exclusion of the normalized Likert scale evaluation	14.79*** [0.00]	30.79*** [0.00]	37.72*** [0.00]	14.37*** [0.00]	18.90*** [0.00]	24.76*** [0.00]	43.96*** [0.00]
F test statistic for exclusion of dummy for same household type	2.98 [0.08]	0.05 [0.82]	1.28 [0.26]	0.35 [0.55]	0.66 [0.42]	0.02 [0.88]	0.03 [0.87]
F test statistic for exclusion of dummy for same material comfort	0.31 [0.58]	3.06 [0.08]	1.79 [0.18]	3.09 [0.08]	0.30 [0.85]	0.96 [0.33]	1.28 [0.26]
F test statistic for exclusion of dummy for same household type and material comfort	1.96 [0.16]	1.96 [0.16]	0.01 [0.91]	1.21 [0.27]	7.56** [0.01]	0.09 [0.77]	0.11 [0.74]

Table 2. Summary of ordinary least squares coefficients and F-tests for exclusion referring to personal characteristics of respondents. Endogenous variable: equivalence scales stated by respondents. Number of observations: 2,042. Standard Errors of coefficients in parentheses. P-values of F-tests in brackets. Boldface characters for coefficients that have P-values below 5%.

*** P<0.001, ** P<0.01, * P<0.05.

Variable	Values	1 adult, 1 child		1 adult, 2 children		1 adult, 3 children	
<i>Region</i>	1: Former East Germany	0.02	(0.01)	0.04*	(0.02)	0.05*	(0.02)
	0: Former West Germany	F=3.11	[0.08]	F=4.47	[0.03]	F=4.81	[0.03]
<i>Gender</i>	1: female	-0.01	(0.01)	-0.00	(0.02)	-0.00	(0.02)
	0: male	F=0.55	[0.46]	F=0.01	[0.92]	F=0.00	[0.96]
<i>Education</i>	1: no degree	0.01***	(0.00)	0.02***	(0.01)	0.03***	(0.01)
	... 6: compl. tech. school/university	F=13.57	[0.00]	F=14.26	[0.00]	F=16.89	[0.00]
<i>Self employed</i>	1: yes	-0.00	(0.02)	-0.00	(0.04)	-0.02	(0.07)
	0: no	F=0.02	[0.90]	F=0.00	[0.98]	F=0.07	[0.80]
<i>Civil servant</i>	1: yes	0.01	(0.03)	0.01	(0.05)	0.01	(0.06)
	0: no	F=0.26	[0.61]	F=0.08	[0.78]	F=0.03	[0.87]
<i>Blue collar</i>	1: yes	-0.01	(0.02)	-0.02	(0.03)	-0.04	(0.04)
	0: no	F=0.13	[0.72]	F=0.53	[0.47]	F=0.85	[0.36]
<i>Pupil, student, trainee</i>	1: yes	0.02	(0.05)	0.06	(0.08)	0.07	(0.11)
	0: no	F=0.20	[0.65]	F=0.75	[0.39]	F=0.50	[0.48]
<i>Working, other</i>	1: yes	0.00	(0.03)	0.01	(0.05)	0.05	(0.08)
	0: no	F=0.01	[0.92]	F=0.11	[0.75]	F=0.57	[0.45]
<i>Pensioner</i>	1: yes	0.00	(0.02)	-0.00	(0.02)	-0.01	(0.03)
	0: no	F=0.08	[0.78]	F=0.01	[0.92]	F=0.04	[0.85]
<i>Unemployed</i>	1: yes	0.01	(0.02)	-0.00	(0.03)	-0.02	(0.04)
	0: no	F=0.22	[0.64]	F=0.01	[0.93]	F=0.29	[0.59]
<i>Housewife/man</i>	1: yes	-0.01	(0.01)	-0.02	(0.02)	-0.03	(0.03)
	0: no	F=0.85	[0.36]	F=1.17	[0.28]	F=1.00	[0.32]
<i>Obligatory military / public service</i>	1: yes	0.03	(0.03)	0.03	(0.04)	0.02	(0.06)
	0: no	F=1.93	[0.17]	F=0.67	[0.41]	F=0.10	[0.75]
<i>Non-working, other</i>	1: yes	0.03	(0.03)	0.04	(0.04)	0.03	(0.06)
	0: no	F=1.88	[0.17]	F=1.18	[0.28]	F=0.38	[0.54]
<i>Number of adults in the respondent's household</i>	1: one adult	0.00	(0.01)	-0.01	(0.02)	-0.02	(0.03)
	2: two adults	F=0.08	[0.78]	F=0.08	[0.78]	F=0.60	[0.44]
<i>Number of children in the respondent's household</i>	0: no children	0.01	(0.01)	0.01	(0.01)	0.02	(0.01)
	... 3: three or more children	F=1.10	[0.30]	F=2.61	[0.11]	F=3.67	[0.06]
<i>Family after-tax income</i>	1: lowest income class	0.00	(0.00)	0.00	(0.00)	0.00	(0.01)
	... 10: highest income class	F=0.06	[0.81]	F=0.04	[0.84]	F=0.01	[0.93]
<i>Age</i>	Age of respondent in years	-0.00	(0.00)	-0.00	(0.00)	-0.00	(0.00)
		F=0.53	[0.47]	F=0.08	[0.77]	F=0.04	[0.85]

Table 2 (continued).

Variable	Values	2 adults, 0 children		2 adults, 1 child		2 adults, 2 children		2 adults, 3 children	
<i>Region</i>	1: Former East Germany	0.04*	(0.02)	0.06*	(0.02)	0.08**	(0.03)	0.10**	(0.04)
	0: Former West Germany	F=5.33	[0.02]	F=6.42	[0.01]	F=8.40	[0.00]	F=7.34	[0.01]
<i>Gender</i>	1: female	-0.01	(0.02)	-0.01	(0.02)	-0.01	(0.03)	-0.01	(0.04)
	0: male	F=0.55	[0.46]	F=0.21	[0.64]	F=0.11	[0.74]	F=0.05	[0.83]
<i>Education</i>	1: no degree	0.01	(0.01)	0.02**	(0.01)	0.03**	(0.01)	0.03**	(0.01)
	... 6: compl. tech. School or university	F=2.54	[0.11]	F=7.52	[0.01]	F=6.88	[0.01]	F=7.54	[0.01]
<i>Self employed</i>	1: yes	0.05	(0.05)	0.04	(0.08)	0.00	(0.10)	-0.03	(0.12)
	0: no	F=0.85	[0.36]	F=0.31	[0.58]	F=0.00	[0.97]	F=0.07	[0.80]
<i>Civil servant</i>	1: yes	-0.02	(0.05)	0.01	(0.06)	0.00	(0.07)	0.00	(0.09)
	0: no	F=0.11	[0.74]	F=0.06	[0.81]	F=0.00	[0.98]	F=0.00	[0.96]
<i>Blue collar</i>	1: yes	-0.01	(0.03)	-0.01	(0.04)	-0.05	(0.05)	-0.08	(0.06)
	0: no	F=0.06	[0.80]	F=0.11	[0.74]	F=0.99	[0.32]	F=1.73	[0.19]
<i>Pupil, student, trainee</i>	1: yes	-0.03	(0.08)	0.06	(0.13)	0.12	(0.15)	0.12	(0.16)
	0: no	F=0.13	[0.72]	F=0.38	[0.54]	F=1.04	[0.31]	F=0.71	[0.40]
<i>Working, other</i>	1: yes	0.04	(0.05)	0.06	(0.07)	0.08	(0.10)	0.11	(0.12)
	0: no	F=0.56	[0.45]	F=1.03	[0.31]	F=1.17	[0.28]	F=1.38	[0.24]
<i>Pensioner</i>	1: yes	0.01	(0.03)	0.01	(0.04)	-0.02	(0.04)	-0.01	(0.05)
	0: no	F=0.05	[0.82]	F=0.08	[0.78]	F=0.18	[0.67]	F=0.01	[0.92]
<i>Unemployed</i>	1: yes	-0.05	(0.03)	-0.04	(0.04)	-0.07	(0.05)	-0.06	(0.07)
	0: no	F=2.77	[0.10]	F=1.21	[0.27]	F=1.86	[0.17]	F=1.02	[0.31]
<i>Housewife/man</i>	1: yes	-0.03	(0.02)	-0.04	(0.03)	-0.05	(0.04)	-0.06	(0.04)
	0: no	F=1.80	[0.18]	F=1.82	[0.18]	F=1.91	[0.17]	F=1.83	[0.18]
<i>Obligatory military / public service</i>	1: yes	-0.01	(0.04)	-0.03	(0.05)	-0.01	(0.07)	-0.01	(0.08)
	0: no	F=0.07	[0.79]	F=0.44	[0.51]	F=0.05	[0.82]	F=0.04	[0.84]
<i>Non-working, other</i>	1: yes	-0.06	(0.04)	-0.04	(0.05)	-0.05	(0.06)	-0.07	(0.08)
	0: no	F=2.00	[0.16]	F=0.66	[0.42]	F=0.63	[0.43]	F=0.82	[0.37]
<i>Number of adults in the respondent's household</i>	1: one adult	0.01	(0.02)	0.01	(0.02)	0.00	(0.03)	0.01	(0.04)
	2: two adults	F=0.31	[0.58]	F=0.20	[0.65]	F=0.01	[0.93]	F=0.03	[0.85]
<i>Number of children in the respondent's household</i>	0: no children	-0.01	(0.01)	-0.01	(0.01)	0.00	(0.02)	0.01	(0.02)
	... 3: three or more children	F=0.69	[0.41]	F=0.77	[0.38]	F=0.00	[0.95]	F=0.23	[0.63]
<i>Family after-tax income</i>	1: lowest income class	-0.00	(0.00)	0.00	(0.01)	0.00	(0.01)	-0.00	(0.01)
	... 10: highest income class	F=0.12	[0.73]	F=0.05	[0.83]	F=0.02	[0.89]	F=0.00	[0.98]
<i>Age</i>	Age of respondent in years	-0.00*	(0.00)	-0.00*	(0.00)	-0.00	(0.00)	-0.00	(0.00)
		F=5.20	[0.02]	F=4.83	[0.03]	F=2.86	[0.09]	F=2.53	[0.11]

Table 3. Summary of seemingly unrelated regressions. Endogenous variable: equivalence scales stated by respondents. Standard errors in parentheses. P-values of F-tests in brackets.

*** P<0.001, ** P<0.01, * P<0.05.

	Germany (835 observations)						
	1 adult, 1 child	1 adult, 2 children	1 adult, 3 children	2 adults, 0 children	2 adults, 1 child	2 adults, 2 children	2 adults, 3 children
Constant	0.99 ^{***} (0.02)	1.03 ^{***} (0.04)	1.09 ^{***} (0.06)	1.27 ^{***} (0.04)	1.26 ^{***} (0.06)	1.30 ^{***} (0.07)	1.36 ^{***} (0.09)
Reciprocal of reference income	271.22 ^{***} (8.70)	482.93 ^{***} (14.83)	698.54 ^{***} (22.10)	215.65 ^{***} (16.25)	460.07 ^{***} (20.27)	674.65 ^{***} (25.43)	886.86 ^{***} (32.62)
Dummy reference income 1,270 Euros	-0.01 (0.01)	-0.01 (0.02)	-0.02 (0.03)	-0.00 (0.02)	-0.01 (0.03)	-0.01 (0.04)	-0.01 (0.05)
Dummy reference income 2,032 Euros	0.01 (0.01)	0.01 (0.02)	0.01 (0.04)	0.03 (0.03)	0.02 (0.03)	0.03 (0.04)	0.02 (0.05)
Dummy reference income 2,794 Euros	-0.00 (0.01)	-0.01 (0.02)	-0.01 (0.04)	-0.02 (0.03)	-0.02 (0.03)	-0.02 (0.04)	-0.02 (0.05)
Adjusted R ²	0.61	0.63	0.62	0.24	0.46	0.53	0.54
F test statistic for exclusion of all reference income dummy variables	0.30 [0.83]	0.30 [0.82]	0.22 [0.88]	0.87 [0.46]	0.54 [0.66]	0.46 [0.71]	0.22 [0.88]

	France (1,115 observations)						
	1 adult, 1 child	1 adult, 2 children	1 adult, 3 children	2 adults, 0 children	2 adults, 1 child	2 adults, 2 children	2 adults, 3 children
Constant	1.03 ^{***} (0.03)	1.07 ^{***} (0.05)	1.08 ^{***} (0.07)	1.26 ^{***} (0.04)	1.26 ^{***} (0.06)	1.25 ^{***} (0.08)	1.24 ^{***} (0.10)
Reciprocal of reference income	234.33 ^{***} (10.56)	437.75 ^{***} (17.86)	621.02 ^{***} (25.08)	202.54 ^{***} (14.63)	411.23 ^{***} (19.94)	604.04 ^{***} (26.93)	786.70 ^{***} (34.67)
Dummy reference income 1,312 Euros	-0.01 (0.02)	-0.02 (0.03)	-0.02 (0.04)	0.00 (0.02)	0.01 (0.03)	0.00 (0.04)	-0.01 (0.05)
Dummy reference income 2,100 Euros	0.01 (0.02)	0.01 (0.03)	0.01 (0.04)	-0.00 (0.02)	-0.00 (0.03)	-0.00 (0.04)	-0.00 (0.05)
Dummy reference income 2,887 Euros	-0.00 (0.02)	-0.00 (0.03)	-0.01 (0.04)	-0.02 (0.02)	-0.01 (0.03)	-0.01 (0.04)	-0.02 (0.06)
Adjusted R ²	0.38	0.42	0.43	0.20	0.35	0.39	0.40
F test statistic for exclusion of all reference income dummy variables	0.43 [0.73]	0.36 [0.78]	0.26 [0.85]	0.21 [0.89]	0.16 [0.92]	0.05 [0.98]	0.04 [0.99]

	Cyprus (650 observations)						
	1 adult, 1 child	1 adult, 2 children	1 adult, 3 children	2 adults, 0 children	2 adults, 1 child	2 adults, 2 children	2 adults, 3 children
Constant	1.08 ^{***} (0.05)	1.19 ^{***} (0.09)	1.28 ^{***} (0.14)	1.24 ^{***} (0.07)	1.31 ^{***} (0.10)	1.43 ^{***} (0.14)	1.52 ^{***} (0.17)
Reciprocal of reference income	192.68 ^{***} (9.22)	351.77 ^{***} (15.89)	519.77 ^{***} (23.82)	168.68 ^{***} (12.35)	321.83 ^{***} (16.84)	499.02 ^{***} (23.29)	661.18 ^{***} (29.20)
Dummy reference income 774 Euros	-0.03 (0.02)	-0.04 (0.04)	-0.07 (0.06)	0.01 (0.03)	-0.02 (0.04)	-0.03 (0.06)	-0.06 (0.07)
Dummy reference income 1,238 Euros	-0.00 (0.02)	0.00 (0.04)	-0.00 (0.06)	0.02 (0.03)	0.01 (0.04)	0.01 (0.06)	0.01 (0.08)
Dummy reference income 1,702 Euros	0.01 (0.02)	0.02 (0.04)	0.02 (0.06)	0.02 (0.03)	0.02 (0.05)	0.03 (0.06)	0.03 (0.08)
Adjusted R ²	0.48	0.51	0.50	0.30	0.45	0.49	0.52
F test statistic for exclusion of all reference income dummy variables	0.76 [0.52]	0.73 [0.53]	0.76 [0.52]	0.15 [0.93]	0.26 [0.85]	0.30 [0.82]	0.40 [0.75]

Table 3 (continued).

India (1,070 observations)							
	1 adult, 1 child	1 adult, 2 children	1 adult, 3 children	2 adults, 0 children	2 adults, 1 child	2 adults, 2 children	2 adults, 3 children
Constant	1.09 ^{***} (0.10)	1.25 ^{***} (0.15)	1.39 ^{***} (0.22)	1.19 ^{***} (0.11)	1.19 ^{***} (0.16)	1.32 ^{***} (0.22)	1.31 ^{***} (0.29)
Reciprocal of reference income	110.65 ^{***} (6.69)	200.92 ^{***} (9.67)	308.39 ^{***} (14.48)	134.11 ^{***} (7.39)	245.18 ^{***} (10.72)	357.38 ^{***} (14.45)	467.95 ^{***} (18.95)
Dummy reference income 552 Euros	-0.01 (0.04)	-0.01 (0.06)	-0.02 (0.09)	0.01 (0.05)	-0.00 (0.07)	-0.02 (0.09)	-0.02 (0.12)
Dummy reference income 967 Euros	-0.02 (0.04)	-0.02 (0.06)	-0.03 (0.09)	-0.02 (0.05)	-0.02 (0.07)	-0.03 (0.09)	-0.04 (0.12)
Dummy reference income 1,381 Euros	0.01 (0.04)	0.01 (0.06)	0.02 (0.10)	0.01 (0.05)	0.01 (0.07)	0.01 (0.10)	0.01 (0.13)
Adjusted R ²	0.28	0.38	0.39	0.31	0.42	0.46	0.47
F test statistic for exclusion of all reference income dummy variables	0.15 [0.93]	0.07 [0.97]	0.08 [0.97]	0.15 [0.93]	0.07 [0.98]	0.09 [0.96]	0.05 [0.99]

China (980 observations)							
	1 adult, 1 child	1 adult, 2 children	1 adult, 3 children	2 adults, 0 children	2 adults, 1 child	2 adults, 2 children	2 adults, 3 children
Constant	1.47 ^{***} (0.11)	1.67 ^{***} (0.22)	1.93 ^{***} (0.37)	1.49 ^{***} (0.12)	1.80 ^{***} (0.20)	2.13 ^{***} (0.31)	2.68 ^{***} (0.44)
Reciprocal of reference income	139.39 ^{***} (8.09)	295.82 ^{***} (16.83)	411.41 ^{***} (27.73)	78.42 ^{***} (9.27)	227.80 ^{***} (15.01)	386.69 ^{***} (23.30)	529.31 ^{***} (33.52)
Dummy reference income 497 Euros	0.03 (0.03)	0.01 (0.06)	-0.05 (0.09)	0.02 (0.03)	-0.02 (0.05)	-0.09 (0.08)	-0.17 (0.11)
Dummy reference income 993 Euros	0.01 (0.03)	-0.04 (0.06)	-0.19 [*] (0.10)	-0.01 (0.03)	-0.08 (0.05)	-0.16 [*] (0.08)	-0.23 [*] (0.12)
Dummy reference income 1,987 Euros	0.00 (0.03)	-0.02 (0.06)	-0.16 (0.10)	-0.03 (0.03)	-0.09 (0.05)	-0.15 (0.09)	-0.19 (0.12)
Adjusted R ²	0.31	0.32	0.27	0.15	0.29	0.32	0.29
F test statistic for exclusion of all reference income dummy variables	0.32 [0.81]	0.24 [0.87]	1.56 [0.20]	0.56 [0.64]	1.10 [0.35]	1.68 [0.17]	1.75 [0.16]

Botswana (477 observations)							
	1 adult, 1 child	1 adult, 2 children	1 adult, 3 children	2 adults, 0 children	2 adults, 1 child	2 adults, 2 children	2 adults, 3 children
Constant	1.40 ^{***} (0.15)	1.56 ^{***} (0.28)	1.61 ^{***} (0.44)	1.15 ^{***} (0.24)	1.47 ^{***} (0.31)	1.56 ^{***} (0.43)	1.75 ^{***} (0.59)
Reciprocal of reference income	115.85 ^{***} (9.75)	233.90 ^{***} (17.48)	351.55 ^{***} (26.97)	122.06 ^{***} (14.57)	249.05 ^{***} (19.01)	388.31 ^{***} (26.73)	527.51 ^{***} (36.21)
Dummy reference income 381 Euros	0.03 (0.04)	0.07 (0.08)	0.10 (0.12)	0.01 (0.07)	0.01 (0.09)	0.03 (0.12)	0.01 (0.16)
Adjusted R ²	0.31	0.32	0.32	0.18	0.33	0.38	0.38
F test statistic for exclusion of all reference income dummy variables	0.69 [0.41]	0.68 [0.41]	0.63 [0.43]	0.01 [0.91]	0.02 [0.88]	0.08 [0.78]	0.01 [0.93]

Figure 1. Scatter plots of stated EIs in Part A of the survey for each RI and each family type.

- 6th degree polynomial fit.
- - - linear regression.

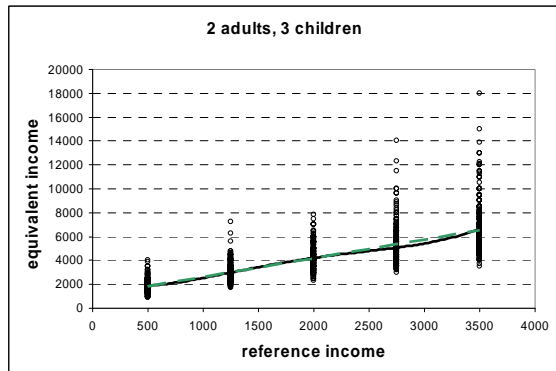
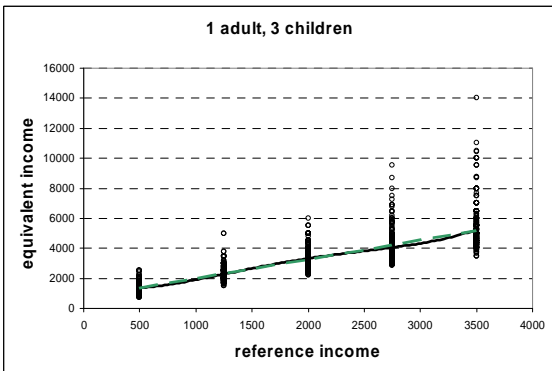
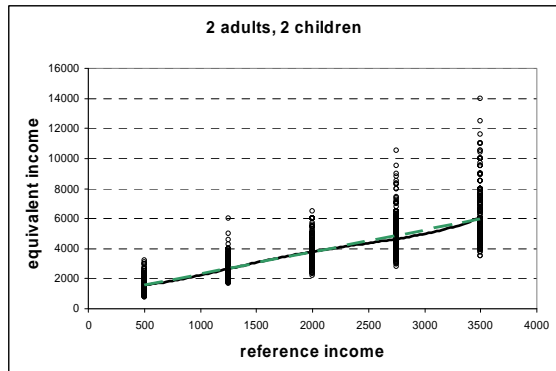
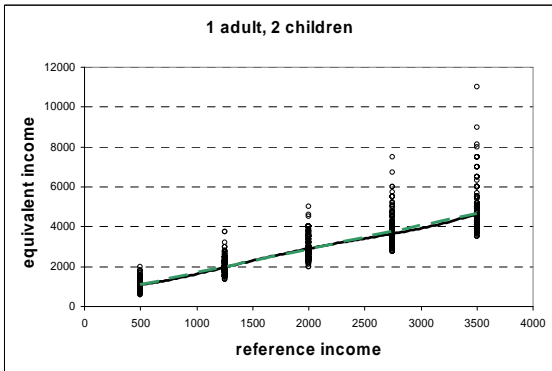
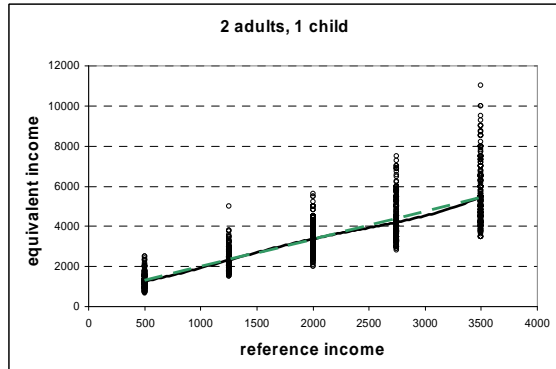
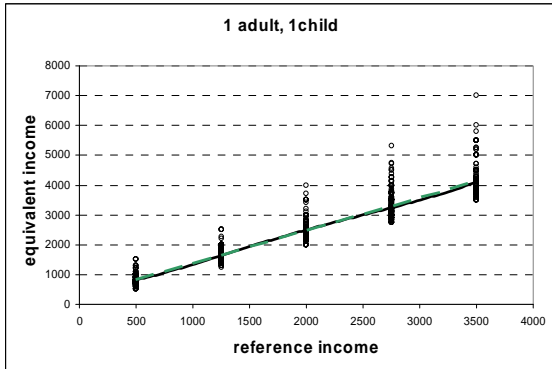
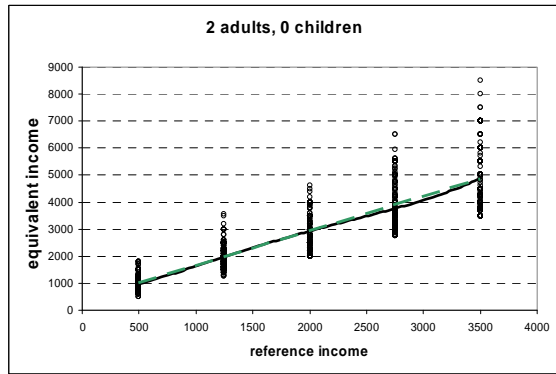


Figure 2. Scatter plots of stated equivalent incomes.
 6th degree polynomial fit

- France
- China
- Germany

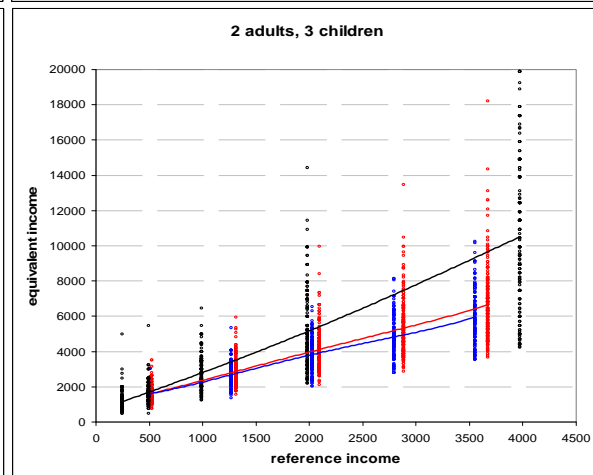
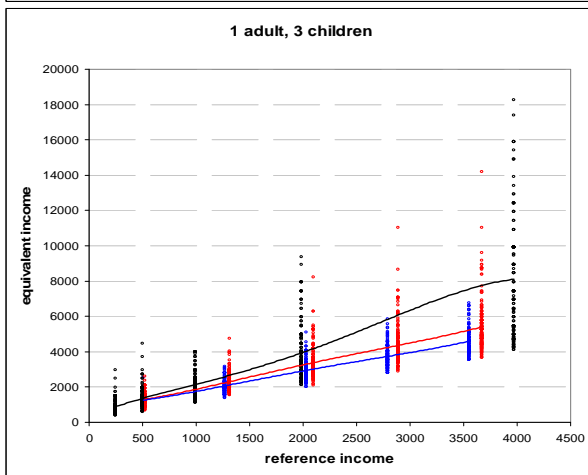
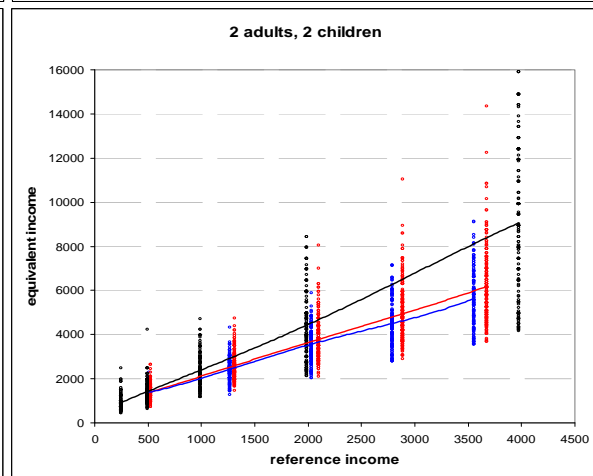
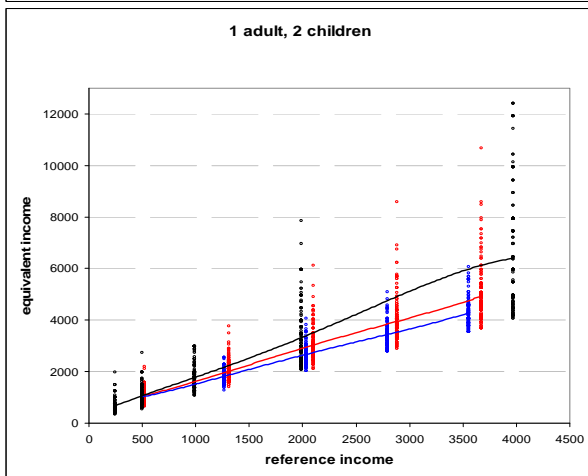
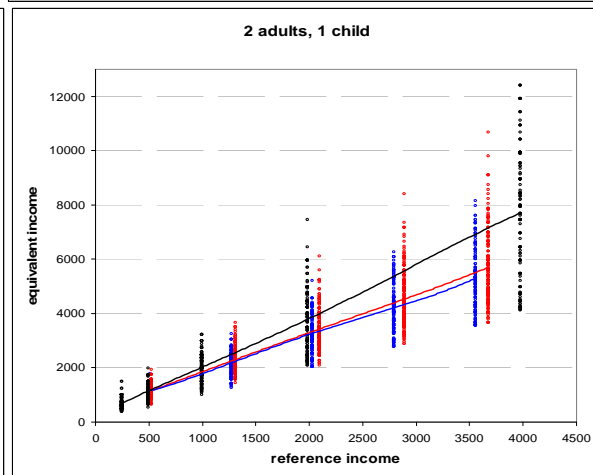
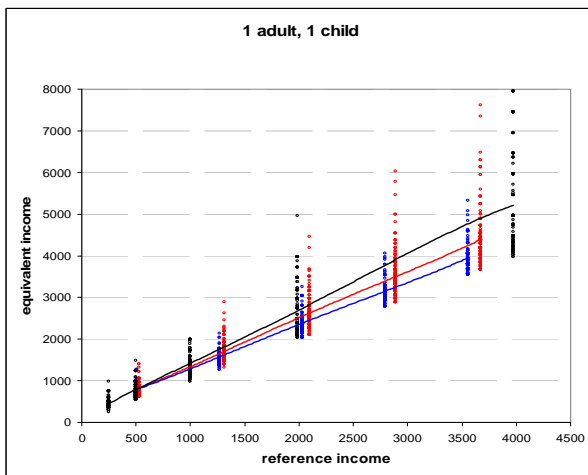
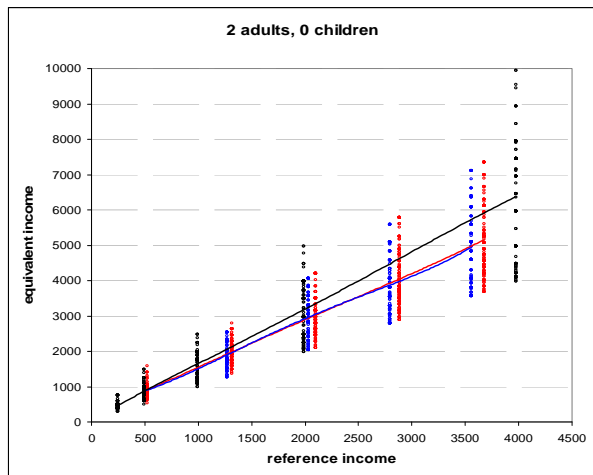
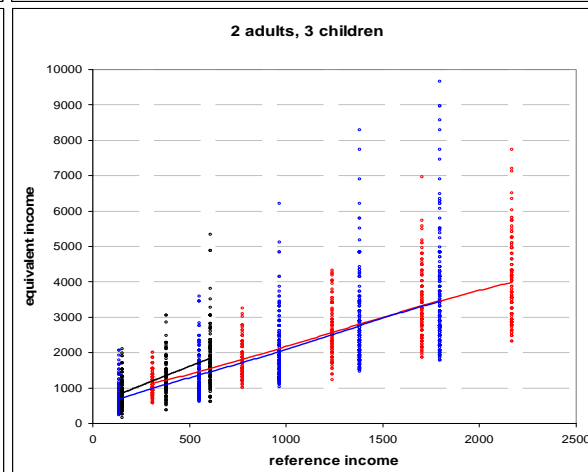
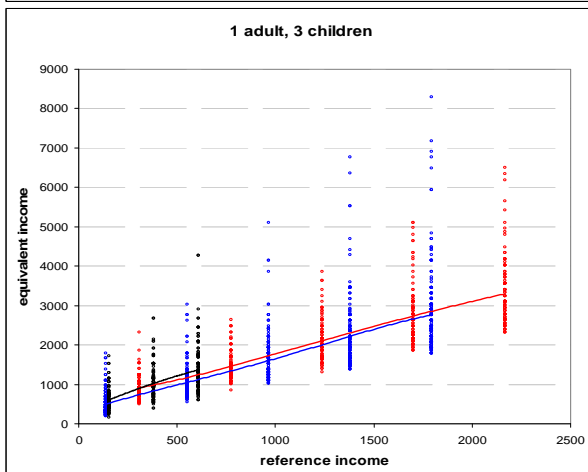
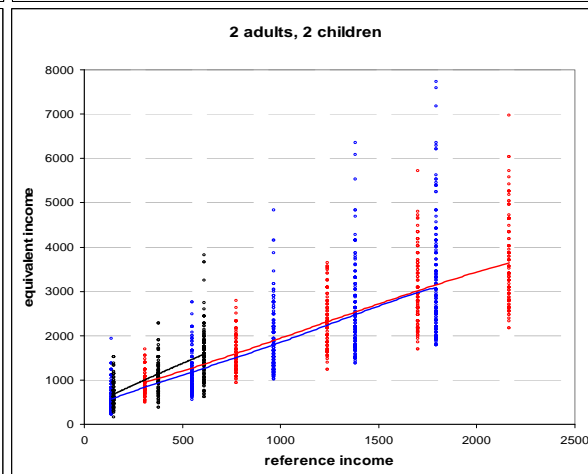
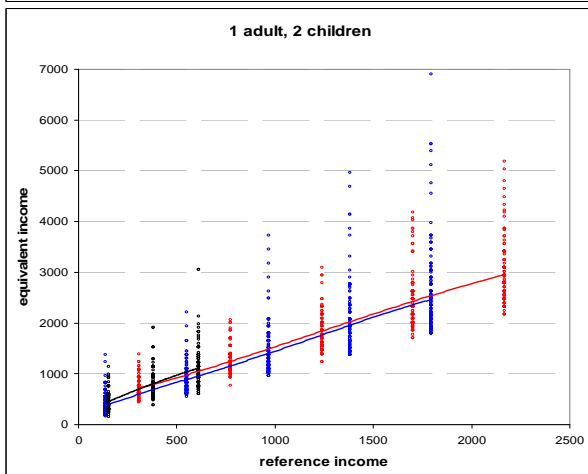
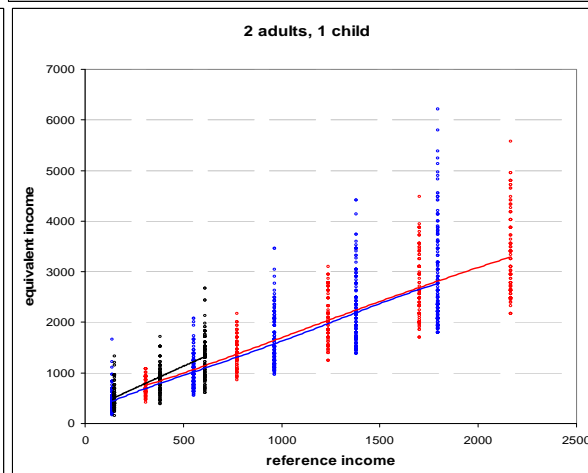
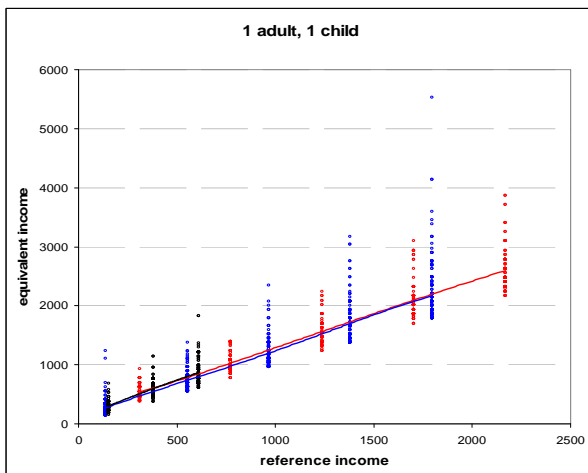
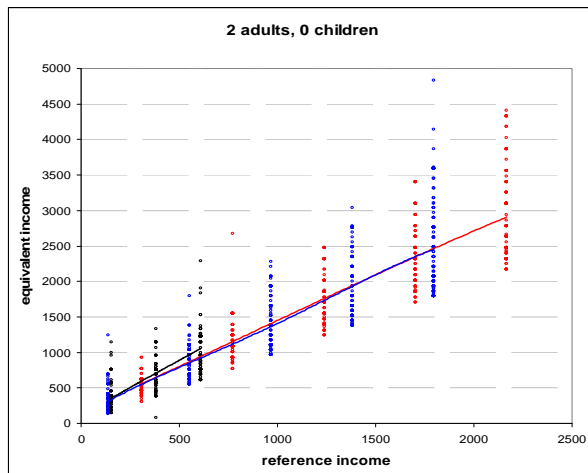


Figure 3. Scatter plots of stated equivalent incomes.
 6th degree polynomial fit

- Cyprus
- Botswana
- India



Additional Appendices (Online) for
Confronting the Representative Consumer with
Household-Size Heterogeneity

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1. Appendix A – Representative Research Sample in Germany: Calibration of Main Evaluation Task and Sampling

In order to implement Part A of the questionnaire efficiently, it is necessary to examine demographic and descriptive income statistics from the country being studied in order to determine appropriate household types and reference incomes (RIs) to use in Part A. In Germany, the eight household types that were chosen represent 86.05% of the overall number of households, as seen in Table A1, based on the most recent German Income and Expenditure Survey (EVS) of 2003.¹ The EVS, provided by the German Statistical Office in five-year intervals, contains representative household-level information on income, wealth, and expenditures for several types of goods. The RIs provided in Part A were determined so as to cover a broad range of the disposable-income distribution for single-childless-adult households in Germany. The amount of EUR 500 per month is the level of total social assistance for a one-member household in Germany. Specifically, the level of monetary social assistance in 2006 for a single, childless adult is EUR 345 per month (see Article 20, Paragraph 2, 2a, 3, Sozialgesetzbuch II (SGB II - “Social Security Code”)).² In addition, households receive housing allowances. The level of housing allowances is contingent upon the rent and also upon the income and wealth of the single, childless adult. A reasonable number is ca. EUR 160. The amount of EUR 1,250 corresponds to the 41st percentile of the one-member-household monthly disposable-income distribution, EUR 2,000 to the 76th, EUR 2,750 to the 89th, and EUR 3,500 to the 94th percentile. Each respondent was provided with only one RI to evaluate in Part A (by random assignment).

¹ See the German Social Science Infrastructure Services at:

http://www.gesis.org/en/social_monitoring/GML/data/inc&exp/index.htm.

² For the German Social Security Code see, http://www.sozialgesetzbuch-bundessozialhilfegesetz.de/_buch/sgb_ii.htm.

The survey's sample consists of 2,042 respondents from all regions of Germany, collected by the research institute "FORSA" ("Gesellschaft für Sozialforschung und statistische Analysen mbH" - Research Institute for Social Research and Statistical Analyses) in 2006. The FORSA institute routinely conducts surveys with a representative online panel of about 10,000 German households. FORSA has stored an extensive set of socioeconomic and demographic variables for each participating household. This enables a pre-screening of respondents' personal and household characteristics. Households were provided with web TVs when internet was not available. Completion times ranged from about 10 to 25 minutes.

The sampling procedure is targeted to obtain enough respondents who live in each of the household types that appear as hypothetical households in Part A. Table A2 shows the breakdown of the large sample from Germany, and Table A3 shows the number of respondents from each family type. Table A3 also compares the percentages of respondents from each household type in the sample with the percentages of household types in the overall German population. This comparison reveals that pre-screening of respondent characteristics is efficient. The household type consisting of 1 adult with 3 children has been more than six times over-represented in the sample compared to the German population. Even so, there were only 19 respondents from households with 1 adult and 3 children. For the other seven household types, respondent numbers are sufficiently high to conduct the tests explained below.

2. Appendix B – Pilot Survey Samples

The breakdown of the samples in pilot studies appears in Tables B1a and B1b. The complete questionnaire appears in Appendix A.1 of Koulovatianos et al. (2005). In Botswana the questionnaire consisted of questions about three reference incomes instead of five. Be-

cause several languages (mainly Setswana and Kalanga, but also Sekgalagadi) are used in Botswana, interviewers had to resort to oral interviews. The response rate with five reference incomes was low and given our planned budget and time constraints we modified the questionnaire so as to increase the response rate. For the purpose of testing the income dependence of equivalence scales three reference incomes serve this task well. For testing the linear relationship between EIs and RIs, three reference incomes are marginally sufficient for such a test. Nevertheless, we include this country in this study as complementary information.

The questionnaire, the sampling strategy and sampling regions for Germany, France, and Cyprus appear in previous studies (see Koulovatianos et al. (2005, 2007)). The sampling region in China was the urban area of Hangzhou and several towns in the province of Zhejiang. In India the sample was collected from cities and villages of three states of south India, Tamil Nadu, Andhra Pradesh, and Karnataka. The cities where our respondents were surveyed are Chennai (Madras) in Tamil Nadu, Hyderabad (Andhra Pradesh), and Bangalore in Karnataka. The questionnaire was provided in the languages of Tamil (Tamil Nadu), Telegu (Andhra Pradesh), in the English language (respondents from Karnataka preferred English instead of our questionnaires provided in the language Kannada) and elderly respondents were given the option of a questionnaire in Hindi. In Botswana sampling was from the capital Gaborone and villages around it. Apart from questionnaires provided in English, a large part of the respondents were interviewed orally, mainly in the languages Setswana and Kalanga. Sample surveys typically lasted between 20-35 minutes, as respondents had to evaluate 5 different RIs.

3. Appendix C – How NLSE suppresses noise from Heterogeneity in Respondent Perceptions of Verbal Characterizations

The existence of a common, “cardinal” perception of verbal characterizations such as “good” or “bad” is not guaranteed.³ This problem can make stated Likert-scale values in Part B noisy across individuals. We have named the source of such noise *Heterogeneity in Respondent Perceptions of Verbal Characterizations (HRPVC)*. To suppress such inter-respondent noise we construct the variable “normalized Likert-scale evaluation” (NLSE).

Table C1 presents the descriptive statistics of Likert-scale values stated in Part B for all household types and RIs. The means and medians across household types for a given reference income are close to each other. This lends support to the results of the pilot survey that was run in advance to define the EIs that were provided in Part B:⁴ respondents of the present survey also perceive the average incomes stated by the respondents of the pilot survey as EIs.

Figure C1 depicts information from the first column of Table C1, which refers to the one-member household. Each box in Figure C1 is defined by the value of the first and third quartile, so each box contains 50% of the values around the median. A dash within a box represents the median response, while each vertical line spans the range of responses. Except for RI = EUR 2,750, the range of responses covers the whole Likert-scale interval that was provided (from 1 to 100). In particular, for the distribution of responses corresponding to RI = EUR 1,250, both the mean and the median lie in the middle of the range, and the two middle quartiles are distanced symmetrically from the median by 20 points in the Likert scale. So, while Figure C1 shows that there is positive correlation between income and

³ See Kahneman and Krueger (2006, pp. 18-21) for a thorough discussion of this difficulty of inter-respondent comparisons of verbal characterizations of well-being.

⁴ These numbers are taken from Koulovatianos et al. (2005) for Germany.

subjective perceptions of living standards, the noisiness of the Likert-scale values indicates the presence of HRPVC. Such noisiness justifies concerns about the effectiveness of using ‘raw’ Likert-scale values for interpersonal comparisons and about their role as conditioning variables in regressions.

The descriptive statistics of NLSE are given by Table C2 and Figures C2 to C6. By the definition of NLSE, noise stemming from HRPVC should be suppressed. Table C2 confirms this suppressive effect of the NLSE.

4. Appendix D – Calibration of the Representative Consumer in Germany in year 2003

In order to calibrate subsistence consumption so as to replicate the numbers appearing in the utility function given by (19) in the paper, we combine equation (16) in the paper with equation (61) appearing in the paper’s Appendix, in order to obtain

$$\beta_j = \psi_{i,j} \cdot \beta_i - \alpha \cdot \chi_{i,j} . \quad (\text{D.1})$$

Setting $i = OMH$, where “ OMH ” denotes a one-member household, and aggregating across all household types, equation (D.1) implies,

$$\sum_{j \in \mathcal{I}} \mu_j \beta_j = \beta_{OMH} \cdot \sum_{j \in \mathcal{I}} \mu_j \psi_{OMH,j} - \alpha \cdot \sum_{j \in \mathcal{I}} \mu_j \chi_{OMH,j} , \quad (\text{D.2})$$

where μ_j is the fraction of households belonging to household type $j \in \mathcal{I}$, in order to obtain the term $\beta^{RC}(t)$ given by equation (13) in the text for year 2003. Data for the vector $\{\mu_j\}_{j \in \mathcal{I}}$ in equation (D.2) is taken from Table A3 of Appendix A, which are taken from the German Income and Expenditure Survey in year 2003. Estimates for the vectors $\{\psi_{OMH,j}\}_{j \in \mathcal{I}}$ and $\{\chi_{OMH,j}\}_{j \in \mathcal{I}}$ are taken from the relevant estimated coefficients in Table 1 of the paper, while $\psi_{OMH,OMH} = 1$, and $\chi_{OMH,OMH} = 0$. Since the estimation appearing in Table 1 of

the paper refers to monthly data, we have multiplied the resulting expression for $\sum_{j \in \mathcal{I}} \mu_j \beta_j$ from equation (D.2) by 12, in order to obtain the utility function referring to one year.

In aggregative models that use the utility function given by (19) in the paper, the appropriate measure of aggregate income to use is one-member-household equivalent income. A distribution of one-member household equivalent incomes (DOMHEI) transforms household-income data referring to different household types into comparable incomes of identical (one-member) households. Because these one-member-household EIs retain the original level of material comfort of each *individual*, they reflect the inequality of living standards among *individuals* in a country.

The construction of a DOMHEI follows this procedure: consider the household income, y^h , of a household which is household type h with n^h members; based on the estimated values of coefficients a^h and b^h in equation (18) in the paper, find the RI that corresponds to y^h , denoted as y_{RI}^h ; assign y_{RI}^h to each household member of that household and include n^h times the income level y_{RI}^h in the DOMHEI. The idea behind the construction of the DOMHEI is to pick each household member from all household types and place him/her into a one-member household (also treating children as adults), providing each individual with the same level of material comfort in this (new) virtual household type as before. The income level y_{RI}^h plays this role of making material-comfort levels equal when transforming all household types into one-member households.

In our application appearing in Figure D1 we have imposed an upper bound on equivalence scales (ESs), equal to the number of household members. This constraint applies when observed household incomes are exceptionally low. Table D1 presents the average ESs based on estimates from Table 1 in the text, imputed in the income distribution for each household type taken from the German Income and Expenditure Survey in year 2003. The expert-based

OECD-modified ESs are presented in the second column of Table D1.⁵ It is evident that our average ESs and these of the OECD differ only slightly, justifying the comparison of the two estimates of the DOMHEI appearing in Figure D1. The fact that our ESs fall with RI, shifts poorer (richer) multi-member households to lower (higher) one-member-household EIs, thickening the resulting density. This thickening impacts inequality of one-member-household EIs substantially: the Gini coefficient increases from 27.37% (OECD ES) to 30.54% with our ESs.

⁵ See the link in the OECD website:

http://www.oecd.org/LongAbstract/0,2546,en_2825_497118_35411112_1_1_1_1,00.html

Tables for Appendices A - D

Table A1. Distribution of household types in Germany. Data refer to the overall population and are taken from the German Income and Expenditure Survey in 2003.

	Household type								
	1 adult, 0 children	1 adult, 1 child	1 adult, 2 children	1 adult, 3 children	2 adults, 0 children	2 adults, 1 child	2 adults, 2 children	2 adults, 3 children	Other
Number of households (in thousands)	14,031.1	931.4	356.3	45.4	11,208.4	2,440.9	2,963.2	808.3	5,312.8
% of population	36.83	2.44	0.94	0.12	29.42	6.41	7.78	2.12	13.95

Table A2. Description of the personal characteristics of the 2,042 respondents in the survey.
^a Respondents who have completed schooling sufficient for general qualification for entrance to a German University; ^b Respondents who stated that they have an occupation, and they either did not state their occupation type, or their occupation type did not fit in the other working categories; ^c Respondents who stated that they are non-working, and they either did not state their status, or their status did not fit in the other categories.

		Number of respondents	% of respondents
<i>Region</i>	Former West Germany	1,541	75.5
	Former East Germany	501	24.5
<i>Gender</i>	Male	465	22.8
	Female	1,577	77.2
<i>Education</i>	No degree	42	2.1
	Basic level of schooling (9 years)	587	28.7
	Secondary School	926	45.3
	Advanced technical college	119	5.8
	High School ^a	163	8.0
	Completed technical school or university	205	10.0
<i>Occupational Status</i>	Self employed	43	2.1
	Civil servant	57	2.8
	White collar	583	28.6
	Blue collar	180	8.8
	Pupil, student, trainee	23	1.1
	Working, other ^b	52	2.5
<i>Status of non-working</i>	Pensioner	327	16.0
	Unemployed	152	7.4
	Housewife/man	452	22.1
	Obligatory military / public service	101	4.9
	Non-working, other ^c	72	3.5
<i>Family after-tax income class</i>	Less than 500 EUR	36	1.8
	Between 500 and 1000 Euros	239	11.7
	Between 1,000 and 1,500 Euros	385	18.9
	Between 1,500 and 2,000 Euros	437	21.4
	Between 2,000 and 2500 Euros	382	18.7
	Between 2,500 and 3,000 Euros	242	11.9
	Between 3,000 and 3,500 Euros	159	7.8
	Between 3,500 and 4,000 Euros	68	3.3
	Between 4,000 and 4,500 Euros	44	2.2
	4,500 Euros or more	50	2.4
<i>Age group</i>	Between 18 and 40 years	863	42.3
	Between 40 and 60 years	831	40.7
	60 years or older	348	17.0
<i>Partner in the household</i>	Yes	1,396	68.4
	No	646	31.6
<i>Number of children in the household</i>	0	860	42.1
	1	521	25.5
	2	491	24.0
	3 or more	170	8.3

Table A3. Distribution of household types of respondents in the survey sample (first two rows). The last row refers to the overall German population, using data from the most recent German Income and Expenditure Survey in 2003. Numbers appearing in the third row are percentages of the sum of households belonging to the eight household types presented in this table.

	Household type							
	1 adult, 0 children	1 adult, 1 child	1 adult, 2 children	1 adult, 3 children	2 adults, 0 children	2 adults, 1 child	2 adults, 2 children	2 adults, 3 children
Number of respondents	445	125	57	19	415	396	434	151
% of respondents	21.79	6.12	2.79	0.93	20.32	19.39	21.25	7.39
% of population in 2003	42.80	2.84	1.09	0.14	34.19	7.45	9.04	2.47

Table A4. Distribution of respondents having an adjusted disposable household income that is similar to the reference income they were asked to evaluate in Part A of the questionnaire. The adjusted disposable household income is the disposable household income divided by the estimated equivalence scale for the respondent's household type. The estimated equivalence scale is the average equivalence scale from responses to Part A.

Respondent's household type	Number of respondents	Percentage of overall sample	Percentage of all respondents who belong to the same household type
1 adult, 0 children	88	4.31	19.78
1 adult, 1 child	26	1.27	20.80
1 adult, 2 children	15	0.73	26.32
1 adult, 3 children	5	0.24	26.32
2 adults, no children	77	3.78	18.55
2 adults, 1 child	77	3.78	19.44
2 adults, 2 children	93	4.55	21.43
2 adults, 3 children	34	1.67	22.52

Table B1a. Breakdown of the samples in Germany, Cyprus, and France

		Germany		Cyprus		France	
		Sample: 167 obs.		Sample: 130 obs.		Sample: 223 obs.	
		N	%	N	%	N	%
<i>Gender</i>	Male	96	57.49	73	56.15	117	52.47
	Female	71	42.51	57	43.85	106	47.53
<i>Partner in the household</i>	Yes	97	58.08	75	57.69	154	69.06
	No	70	41.92	55	42.31	69	30.94
<i>Living with parents</i>	Yes	---	---	37 ^a	28.46	---	---
	No	---	---	93	71.54	---	---
<i>Number of children in the household</i>	0	123	73.65	82	63.08	102	45.74
	1	18	10.78	18	13.85	45	20.18
	2	15	8.98	23	17.69	46	20.63
	3 or more	11	6.59	7	5.38	30	13.45
<i>Family after-tax income class</i>	1	32	19.16	9	6.92	18	8.07
	2	44	26.35	25	19.23	30	13.45
	3	37	22.16	24	18.46	41	18.39
	4	37	22.16	31	23.85	49	21.97
	5	17	10.18	41	31.54	85	38.12
<i>Occupational group</i>	Welfare recipient or unemployed	7	4.19	2	1.54	7	3.14
	Blue-collar worker	10	5.99	2	1.54	6	2.69
	White-collar worker	83	49.70	40	30.77	48	21.52
	Civil servant	13	7.78	40	30.77	29	13.00
	Pupil, student, trainee	34	20.36	30	23.08	102	45.74
	Self-employed	7	4.19	13	10.00	13	5.83
	Pensioner	10	5.99	0	0.00	6	2.69
	Housewife, -man	3	1.80	3	2.31	12	5.38
<i>Education</i>	Below 9 years of education	1	0.60	4	3.08	0	0.00
	Completed Extended Elementary School	21	12.57	8	6.15	13	5.83
	Completed Special Secondary School	39	23.35	---	---	43	19.28
	Completed Secondary School Technical	65	38.92	65	50.00	37	16.59
	School/University degree	41	24.55	53 ^b	40.77	130	58.30
	0	31	18.56	9	6.92	37	16.59
	1	55	32.93	34	26.15	72	32.29
<i>Number of siblings during childhood</i>	2	47	28.14	40	30.77	59	26.46
	3 or more	34	20.36	47	36.15	55	24.66

Note. The threshold of the first "family-after tax income class" is the country-specific poverty line for a single childless adult. Then, we add increments such that the mean of the third income class is about the mean household income in the respective country. The breakdown has already appeared in Koulovatianos et al. (2005).

^a One of the respondents who were living with their parents also had a partner and two children.

^b 14 out of the 53 highly educated respondents in Cyprus had finished a technical school (3 years of higher education).

Table B1b. Breakdown of the samples in China, India, and Botswana

		Botswana		China		India	
		Sample: 159 obs.		Sample: 196 obs.		Sample: 214 obs.	
		N	%	N	%	N	%
<i>Gender</i>	Male	70	44.03	130	66.33	136	63.55
	Female	89	55.97	66	33.67	78	36.45
<i>Partner in the household</i>	Yes	89	55.97	146	74.49	---	---
	No	70	44.03	50	25.51	---	---
<i>Number of adults in the household</i>	1	---	---	---	---	12	5.61
	2	---	---	---	---	73	34.11
	3	---	---	---	---	35	16.36
	4	---	---	---	---	56	26.17
	5	---	---	---	---	22	10.28
	6	---	---	---	---	10	4.67
	7 or more	---	---	---	---	6	2.80
<i>Number of children in the household</i>	0	48	30.19	159	81.12	74	34.58
	1	26	16.35	27	13.78	48	22.43
	2	40	25.16	7	3.57	62	28.97
	3 or more	45	28.30	3	1.53	30 ^a	14.02
<i>Family after-tax income class</i>	1	10	6.29	42	21.43	4	1.87
	2	18	11.32	47	23.98	22	10.28
	3	48	30.19	56	28.57	24	11.21
	4	42	26.42	32	16.33	39	18.22
	5	41	25.79	19	9.69	37	17.29
	6	---	---	---	---	88	41.12
<i>Occupational group</i>	Welfare recipient or unemployed	30	18.87	4	2.04	8	3.74
	Blue-collar worker	19	11.95	11	5.61	26	12.15
	White-collar worker	24	15.09	5	2.55	41	19.16
	Civil servant	53	33.33	5	2.55	23	10.75
	Pupil, student, trainee	15	9.43	140	71.43	54	25.23
	Self-employed	13	8.18	28	14.29	42	19.63
	Pensioner	2	1.26	0	0.00	9	4.21
	Housewife, -man	3	1.89	3	1.53	8	3.74
	Farmer	---	---	---	---	3	1.40
<i>Education</i>	No schooling	---	---	4	2.04	1	0.47
	Basic schooling	5	3.14	16	8.16	3	1.40
	Completed Primary School	7	4.40	9	4.59	15	7.01
	Completed Junior High School	21	13.21	13	6.63	44	20.56
	Completed High School Technical	39	24.53	147	75.00	93	43.46
	School/University degree	87	54.72	7	3.57	58	27.10
<i>Number of siblings during childhood</i>	0	31	19.50	71	36.22	33	15.42
	1	20	12.58	58	29.59	52	24.30
	2	27	16.98	35	17.86	47	21.96
	3 or more	81	50.94	32	16.33	82	38.32
<i>Age group</i>	Less than 20	---	---	---	---	49	22.90
	Between 20 and 40	---	---	---	---	127	59.35
	40 or more	---	---	---	---	38	17.76
<i>Living area</i>	Urban	107	67.30	104	53.06	190	88.79
	Rural	52	32.70	92	46.94	24	11.21

Note. The threshold of the first "family-after tax income class" is the country-specific poverty line for a single childless adult. Then, we add increments such that the mean of the third income class is about the mean household income in the respective country.

^a In India. 8 households have 4 children. 2 households have 5 children, 3 households have 6 or more children.

Table D1. Average equivalence scales. Equivalence scale estimates taken from the regression in Table 1 in the text depend on the level of material comfort. These equivalence scale estimates are used in order to construct a distribution of one-member households' equivalent incomes from the (most recent) German Income and Expenditure Survey in year 2003. The averages of the equivalence scales imputed in the German Income and Expenditure Survey income distribution (for each household type) are reported in the first column of this table.

Household type	Average equivalence scales from the estimates of the present study	OECD-modified equivalence scale
1 adult, 1 child	1.32	1.30
1 adult, 2 children	1.55	1.60
1 adult, 3 children	1.83	1.90
2 adults, 0 children	1.64	1.50
2 adults, 1 child	1.83	1.80
2 adults, 2 children	2.04	2.10
2 adults, 3 children	2.29	2.40

Figures for Appendices C and D

Figure C1. Box plots of stated Likert-scale values for the reference household.

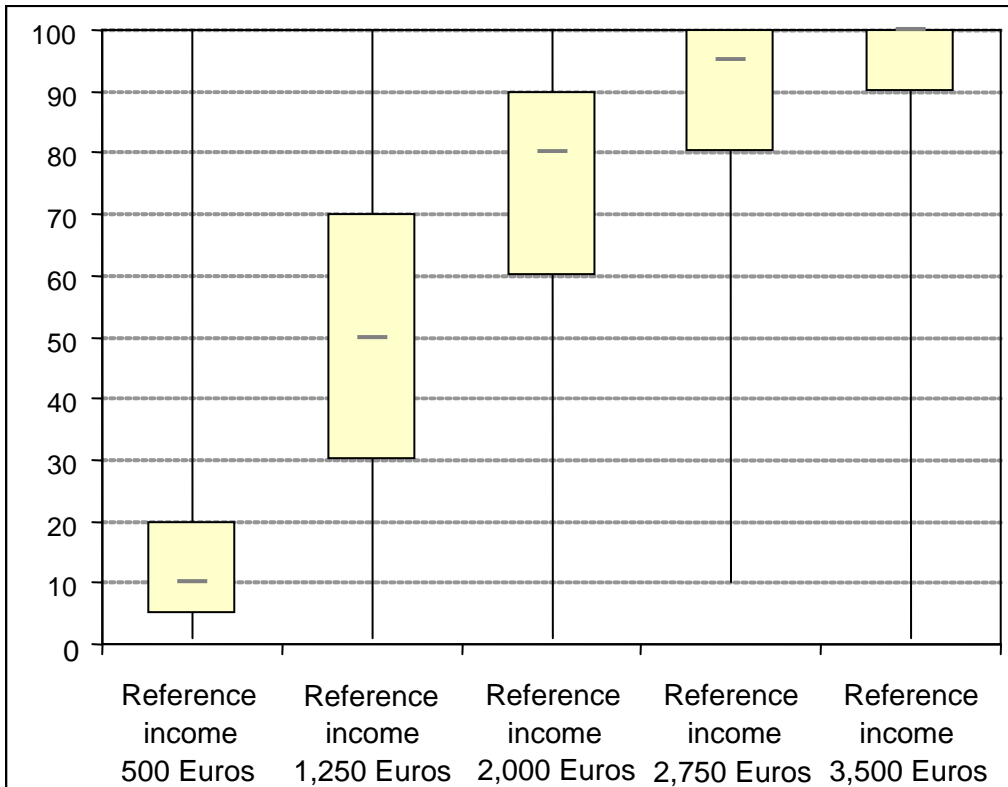


Figure C2. Box plots of Normalized Likert-scale Evaluations for different household types at a reference income of 500 Euros.

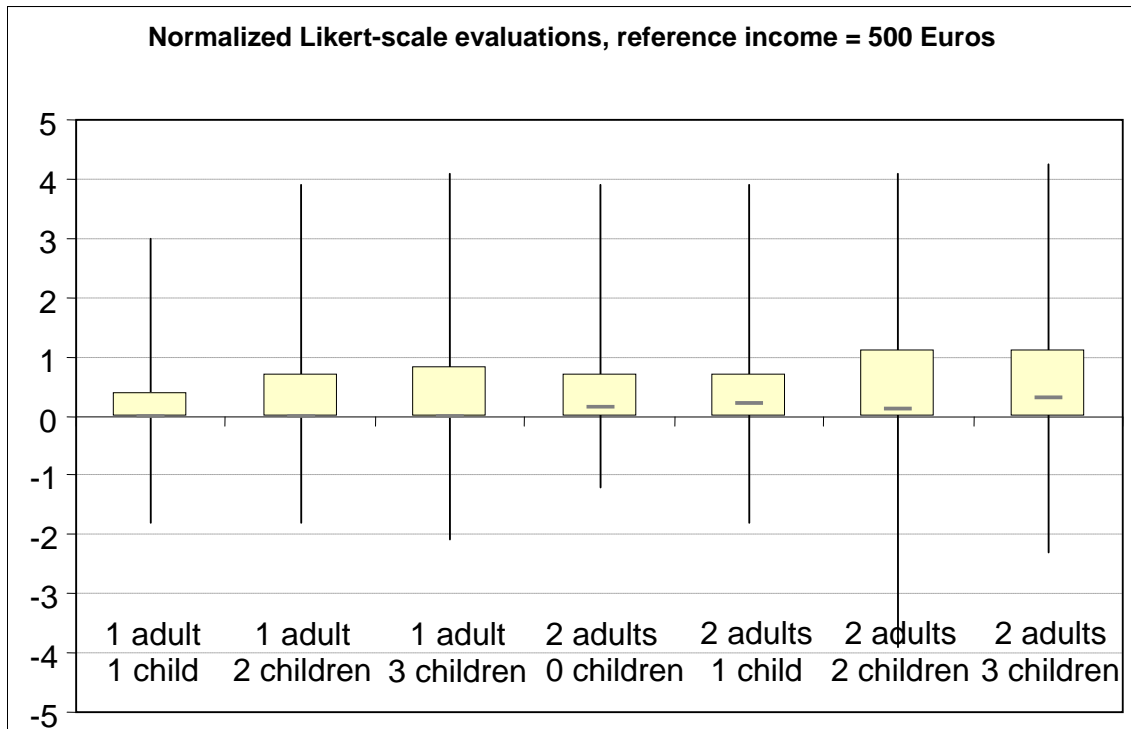


Figure C3. Box plots of Normalized Likert-scale Evaluations for different household types at a reference income of 1,250 Euros.

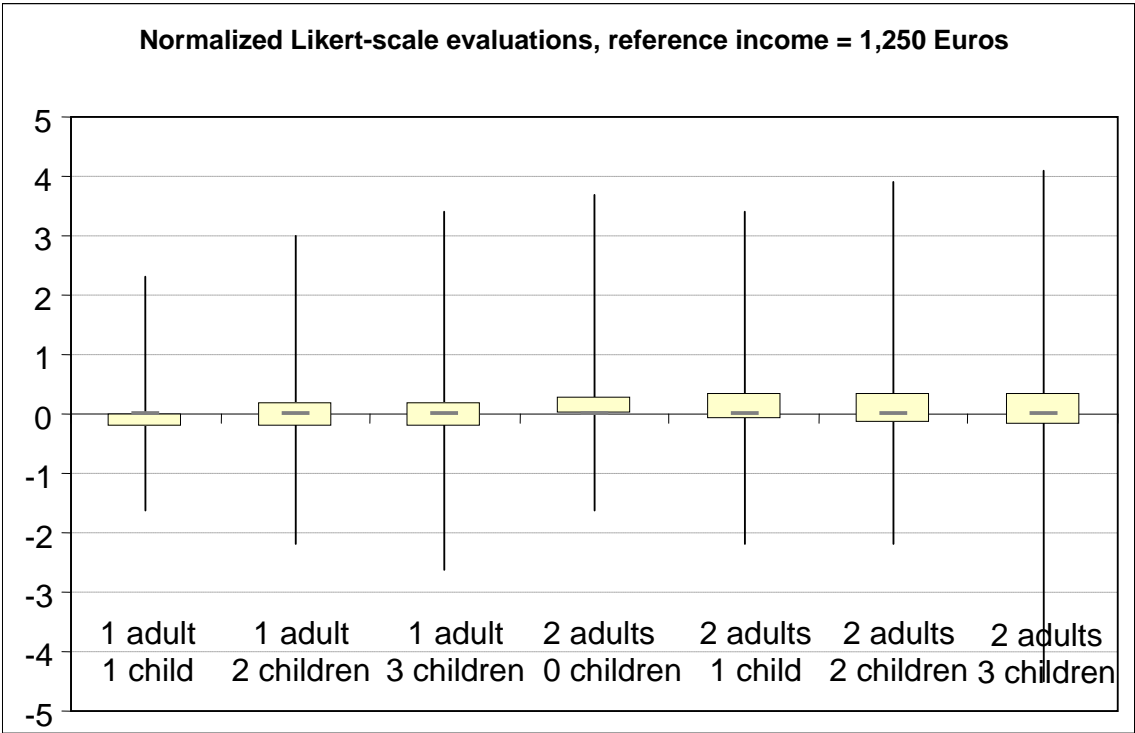


Figure C4. Box plots of Normalized Likert-scale Evaluations for different household types at a reference income of 2,000 Euros.

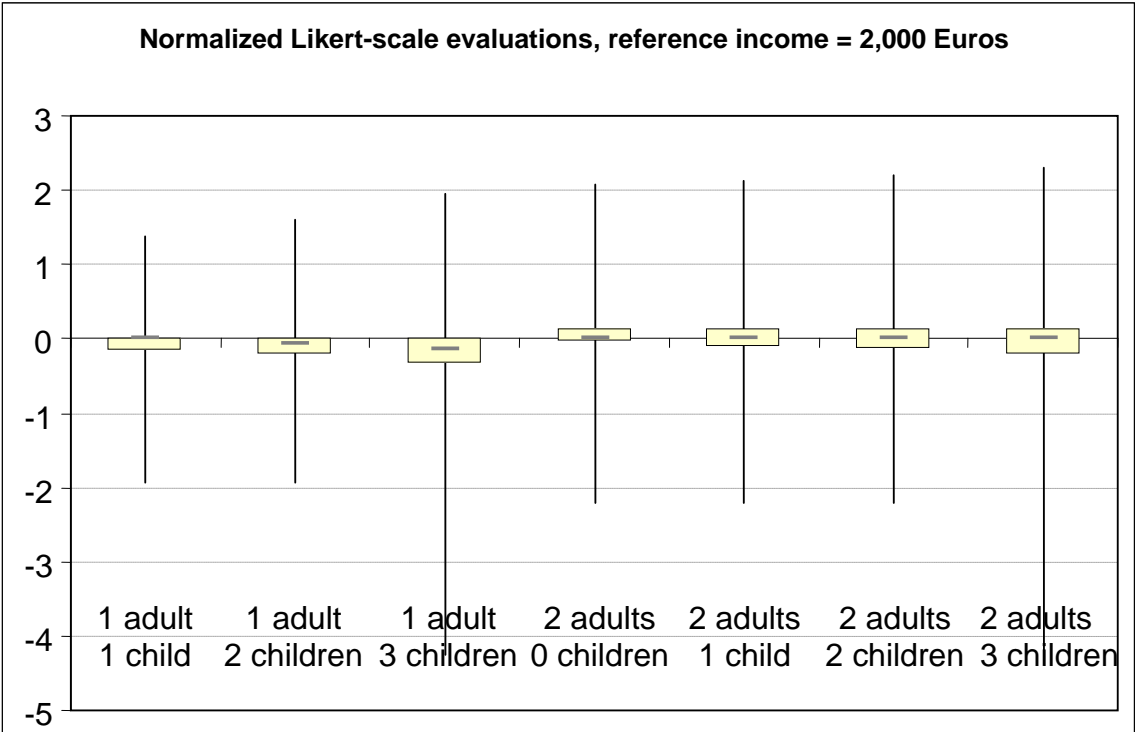


Figure C5. Box plots of Normalized Likert-scale Evaluations for different household types at a reference income of 2,750 Euros.

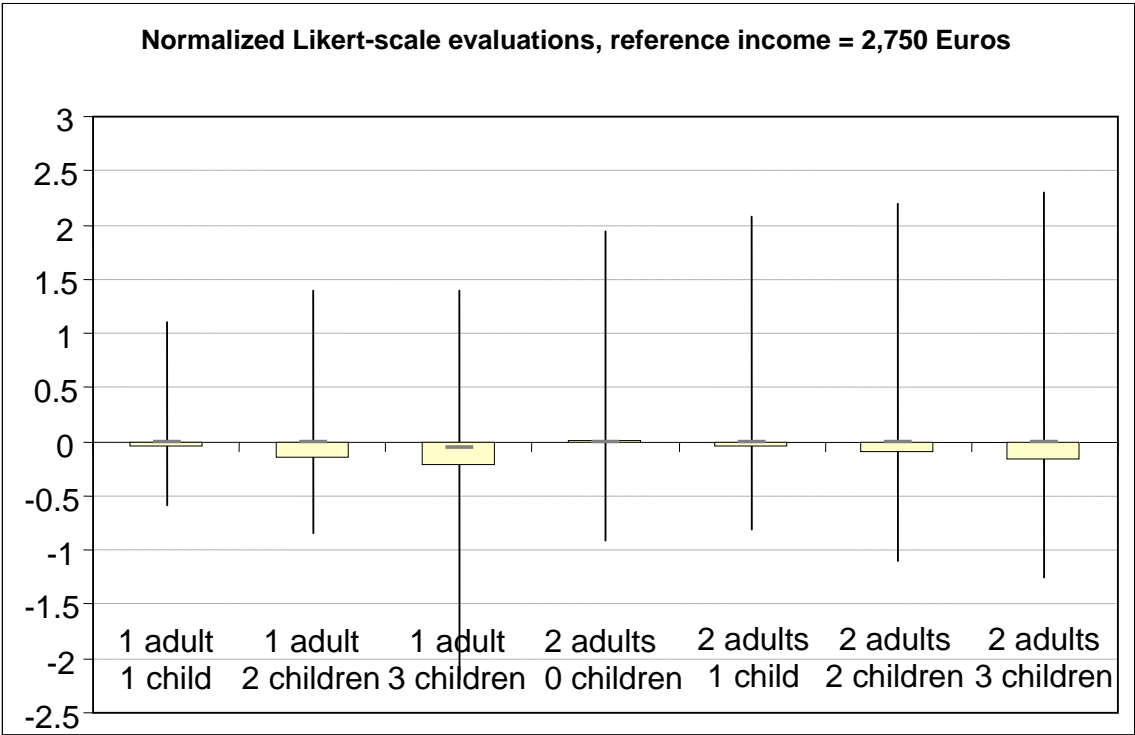


Figure C6. Box plots of Normalized Likert-scale Evaluations for different household types at a reference income of 3,500 Euros.

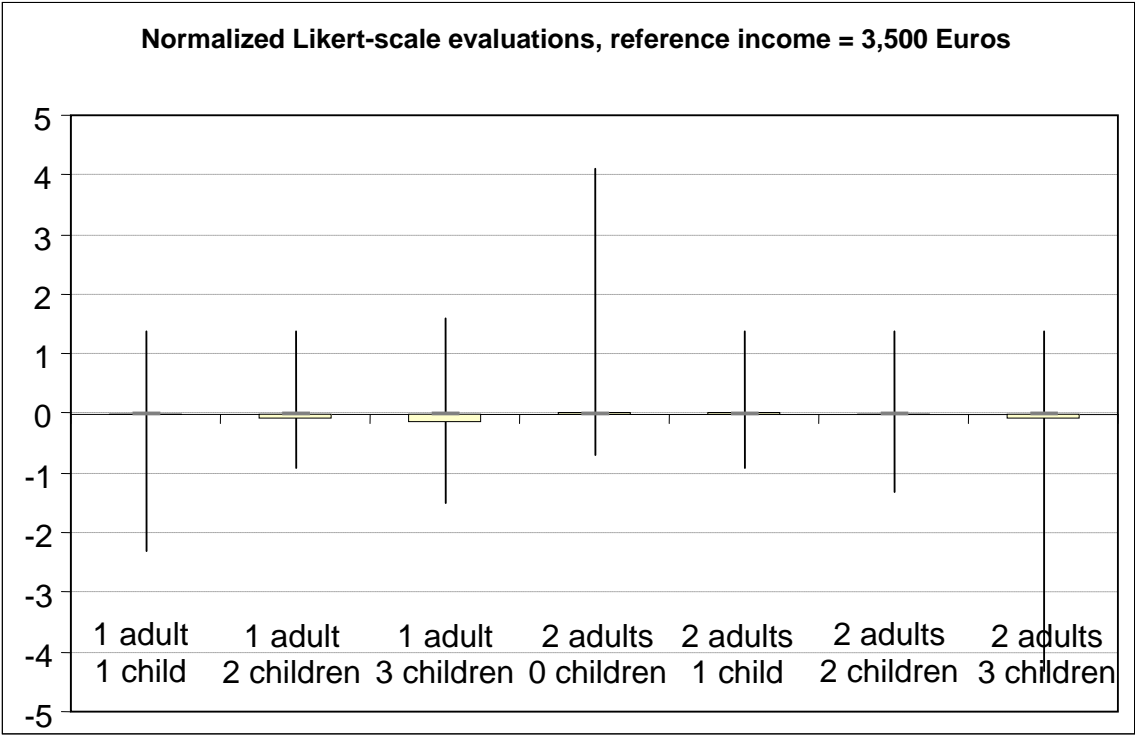
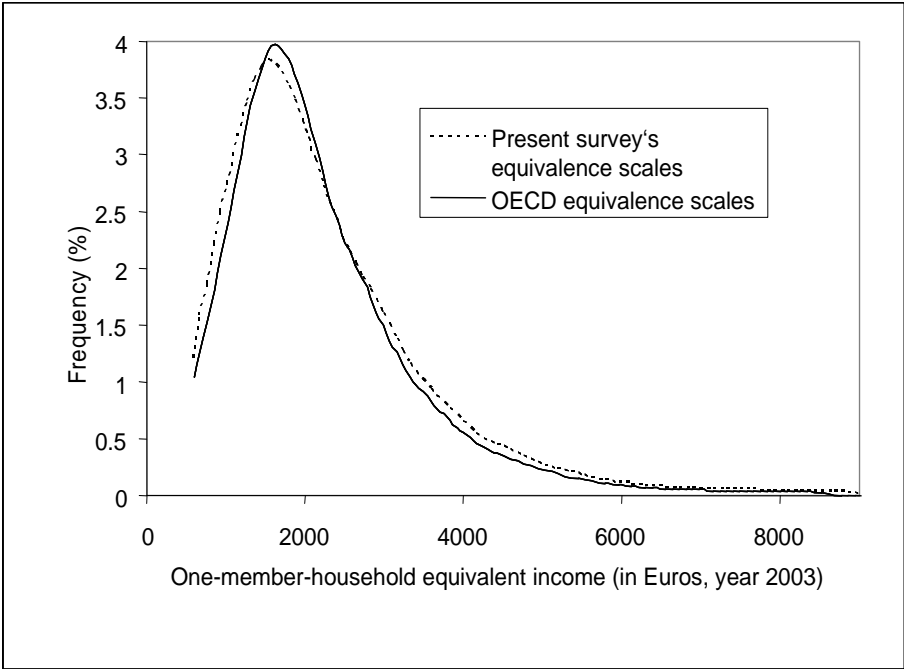


Figure D1. Distribution of one-member-household equivalent incomes calculated using the OECD-modified equivalence scales and equivalence-scale estimates taken from the regressions in Table 1 from the present survey. Household-income data are taken from the German Income and Expenditure Survey 2003.



Appendix E

Survey Instrument Documentation

Information on the connection between a household's demographic composition and the level of material comfort that its income can buy for its members is important for researchers in diverse disciplines. This survey instrument is designed so as to obtain direct estimates of this connection from respondents.

The survey was implemented in automated and electronic form by a professional research institute, FORSA ("Gesellschaft für Sozialforschung und statistische Analysen mbH" – Research Institute for Social Research and Statistical Analyses). Each participating household was equipped with a "set-top-box" that provided Internet access and that was linked to the household's television set.

An introduction addressed to respondents provides a short explanation of the survey topic and a clarification of the concepts that follow. The actual questionnaire consists of two Parts, Part A and Part B. Part A contains the main evaluation task: to provide incomes that equalize the level of material comfort across different hypothetical household types. Part B poses the same assessment problem as in Part A, but using a different means of communication. Respondents are asked to assess the material comfort of different hypothetical household types with specific income levels on Likert scales.

Key advantages of the survey instrument:

- Direct assessments of incomes that equalize the level of material comfort of different household types, enabling the quantification of household-size economies.
- Posing the same evaluation problem using different means of communication in Parts A and B allows for a test of the effectiveness of the survey instrument, suggested in Part A.
- Relevance of the main evaluation task with observable characteristics of the respondent enables a test of effectiveness of the survey instrument. The socio-economic and demographic composition of the respondent's household, may limit her/his available information and ability to evaluate hypothetical household types and levels of material comfort, thus contaminating the results due to a limited-information bias. Comparing answers from respondents whose socio-economic and demographic characteristics are close to those of the hypothetical households they examine with answers from all other respondents enables a test for limited-information bias.
- Low respondent burden: respondents can complete the questionnaire (Introduction, Parts A and B) in about 10-25 minutes.
- High flexibility: Parts A and B can be adjusted easily so as to encompass other hypothetical household types and levels of material comfort.

Introduction for the respondents

Purpose of the survey

In general, different household types may need different incomes in order to attain the same level of material comfort. Since assessing such incomes in an objective way is difficult, we would like to ask you for your personal evaluation of these incomes for a number of different household types. Please note that in this questionnaire there are no “right” or “wrong” answers. So, your answers should only reflect your personal judgements.

[Technical note to the researcher. Respondents click a button to switch to the next screen.]

Instruction

You will frequently read the expression “monthly net household income.” Such a “monthly net household income” is the income amount a household has at its disposal after paying taxes and social security contributions (health insurance contributions, compulsory long term care insurance contributions, unemployment insurance contributions, and contributions to the pension system).

“Monthly net household income” encompasses:
Salary and earnings,
Income from being self-employed,
Pensions,
Unemployment benefits and social benefits,
Accommodation allowance,
Child allowances,
Incomes from rent and lease, and
Other incomes such as returns on investment, interest, etc.

[Technical note to the researcher. Respondents click a button to go to the next screen.]

PART

A

Now, please think about a situation where a single, childless adult has a monthly net household income of 500 Euros.

In this survey, there are seven other household types:

with 1 adult and 1 child

with 1 adult and 2 children

with 1 adult and 3 children

with 2 adults and no children

with 2 adults and 1 child

with 2 adults and 2 children

with 2 adults and 3 children

Assume that adults are ages 35 to 55 and children are ages 7 to 11.

[Technical note to the researcher. Respondents click a button to go to the next screen.]

Which monthly net household income would each of these seven household types need in order to attain the same level of material comfort as the single, childless, adult household with the monthly net household income of 500 Euros?

You should state this monthly net household income for each household type in the table that will follow on the next screen. Please note that your answers should reflect only your personal judgements.

[Technical note to the researcher. Respondents click a button to go to the next screen.]

Which monthly net household income would each household type need in order to attain the same level of material comfort as the single, childless, adult household with the monthly net household income of 500 Euros?

Please state income amounts in Euros.

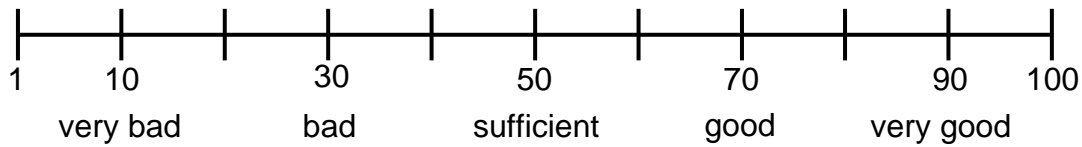
1 adult without children	500 Euros
1 adult, 1 child	
1 adult, 2 children	
1 adult, 3 children	
2 adults, no children	
2 adults, 1 child	
2 adults, 2 children	
2 adults, 3 children	

[Technical note to the researcher. The reference income level provided in the table is randomly assigned to the respondents. If a respondent does not report an income amount for a household type, there is a reminder: "please fill in income amounts in all empty cells of the table." If a respondent's entries are not numbers, there is a reminder: "please state numbers only." If a respondent states income amounts that are decreasing inversely with household size, a box opens: "Usually, larger household types also need higher incomes in order to attain a specific living standard. Please, make sure that you are not stating how much income should be added compared to a smaller household type, but how much the total net household income should be. Please make sure that the entries you made are indeed total net household incomes." This box opens only once, and its intention is to reduce misunderstandings by respondents. However, if a respondent did not adjust the entries she/he made in the table, she/he was free to do so. Respondents click a button to go to the next screen.]

PART B

We will show you several household types with a given monthly net household income. Please evaluate the material comfort that these monthly net household incomes bring to the different household types on a scale ranging from 1 to 100 points. The values of this scale have the following meaning:

Level of material comfort



Please complete the following table by evaluating the monthly net income of each household type on the scale of 1-100.

All values between 1 and 100 are permissible.

	Level of material comfort (in points)
1 adult, no children with 3,500 Euros	
1 adult, 1 child with 3,900 Euros	
1 adult, 2 children with 4,200 Euros	
1 adult, 3 children with 4,550 Euros	
2 adults, no children with 4,850 Euros	
2 adults, 1 child with 5,250 Euros	
2 adults, 2 children with 5,550 Euros	
2 adults, 3 children with 5,850 Euros	

[Technical note to the researcher. The numbers provided in this table are estimates of average equivalent incomes for five reference income levels from an independent study. The five reference incomes are the same as the reference income levels in Part A. So, altogether, five profiles of equivalent incomes (including a reference income for the single, childless, adult household) were evaluated by the survey sample, one profile per respondent. One out of these five equivalent-income profiles was randomly assigned to a respondent. If a respondent reports less than eight Likert scale values, there is a reminder: "please fill in all empty cells of the table." If a respondent's answers do not fall in the given range of the Likert scale (1-100), there is a reminder to "please state numbers between 1 and 100 only."]