Perturbative Quantum Gravity.

Abstract: The subject of perturbative quantum gravity is currently undergoing a renaissance period. Partly this has to do with new tools being used, partly with new mathematical structures that are being explored. This talk is an overview of some of these developments, prepared for a mathematical audience. No knowledge of physics is assumed.

I will start by introducing the main characters: Einstein-Hilbert action and its perturbative expansion; perturbative quantization and Feynman diagrams; correlation functions and the graviton scattering amplitudes. I will then review some modern developments such as: (i) analytic continuation to complex momenta and recursion relations for amplitudes; (ii) "scattering equations" on an n-punctured sphere and a new explicit formula for the tree-level graviton scattering amplitudes; (iii) gauge-theoretic formulation of gravity and new gravitational perturbation theory.
Duality for n-fold vector bundles.

Abstract: Double vector bundles have been implicit in differential geometry for many years: for a vector bundle \( A \to M \) the tangent \( TA \) with its two structures (over \( A \) and over \( TM \)) gives the formulation of connections in \( A \) which was used by Dieudonné, and \( T(TM) \), \( T(T^*M) \) and \( T^*(T^*M) \) are basic in geometric mechanics. In general a double vector bundle is a manifold \( D \) with two compatible vector bundle structures over bases \( A \) and \( B \) which are themselves vector bundles over a common manifold \( M \). Such a \( D \) can be dualized in two ways and these duals are themselves dual over a base which emerges from the double structure. The two dualizations generate the symmetric group of order 6.

For triple vector bundles, Gracia-Saz and the speaker proved that the corresponding group is of order 96 and is a non-split extension of \( S_4 \) by the Klein 4-group. For \( n \)-fold vector bundles the corresponding group is an extension of \( S_{n+1} \) by a direct product of cyclic groups of order 2. It is a subgroup of \( O(k, \mathbb{Z}) \) where \( k \) depends on \( n \), but is not a Coxeter group.

I will describe the pairing of the two duals of a double vector bundle which is the basis of this work, and the results for \( n = 3, 4 \). If time permits I will describe the interpretation – on which further developments rest – of the kernel elements as certain Euler graphs.

This work arose in the study of bracket structures associated to Poisson manifolds and Lie algebroids, but the talk requires no knowledge of these. An acquaintance with the concept of vector bundle is sufficient.
References


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Generalized polynomials and functional equations.

Abstract: Solutions \( f \) of a certain type of functional equations have the property that \( \Delta_{h_1} \circ \Delta_{h_2} \circ \ldots \circ \Delta_{h_{n+1}} f = 0 \) for all \( h_1, h_2, \ldots, h_{n+1} \), where \( \Delta_h f \) is defined by \( (\Delta_h f)(x) := f(x + h) - f(x) \). One possibility to define generalized polynomials (of degree at most \( n \)) is by saying that \( f \) has to satisfy the equation above.

Here other possible definitions and relation between them are presented.
Tone rows and tropes.

Abstract: Complete classification of tone rows in the twelve tone scale is described. We analyze the notion of “equivalent” or “similar” tone rows and, therefore, we introduce the notion of “group actions”. Equivalent tone rows are collected to an orbit of tone-rows. Moreover, we deduce that different degrees of similarity can be described by different groups $G$ acting on the set of all tone rows. In the sequel we explain which groups $G$ are musically reasonable to be taken into account. The main objects of the present talk are the orbits of tone rows under the action of the direct product of two dihedral groups, i.e. $G = D_{12} \times D_{12}$. This means that tone rows are equivalent if and only if they can be constructed by transposing, inversion, retrograde, and/or time shift from a single row. Consequently, tone rows which are equivalent according to the notion of Arnold Schönberg are equivalent in our sense. However, since we also consider the time shift, there exist tone rows which are not equivalent according to Schönberg but equivalent according to our notion.

For some tone rows $f$ it is possible that certain operations leave $f$ invariant. This leads to the notion of “stabilizers” and “stabilizer types” of tone rows. In our main setting of $D_{12} \times D_{12}$-orbits of tone rows we distinguish 17 different stabilizer types.

Computing the list of intervals between consecutive tones of a tone row, we determine the interval structure of $D_{12} \times D_{12}$-tone rows, we discuss all-interval rows and all-distances-twice rows.
The notion of “tropes” introduced by Josef Matthias Hauer suits very well for the classification of $D_{12} \times D_{12}$-tone rows. We describe tropes as unordered pairs of hexachords and determine $G$-orbits of tropes for certain groups $G$. The trope structure is an interesting property of the $D_{12} \times D_{12}$-orbits of tone rows. It can be used to describe the different stabilizer types in our main setting.

Finally, a database containing the complete list of $D_{12} \times D_{12}$-orbits of tone rows is presented. Next to many different “properties” of tone rows we were collecting tone rows appearing in works of various composers. Hence it is also possible to search for musical information on a given tone row. This opens the door for new research: Since we have normal forms of tone rows, it is easy to check, whether similar tone rows appeared in different compositions. Or knowing certain properties of tone rows it is interesting to study whether we can deduce from the composition that the composer was aware of these properties. (Joint work with Peter Lackner, University of Music and Performing Arts Graz.)
Stochastic calculus for fractional Brownian motion in Brownian time.

Abstract: Fractional Brownian motion has been widely used to model a number of phenomena in diverse fields such as biology, climatology, or finance to name a few. In this talk, I will introduce a related and new process, the so-called fractional Brownian motion in Brownian time (FBMBT). In a first part, I will explain why I have been interested in studying such a process. One of my motivations comes from the desire to find a stochastic model for a crack filled with a gas. FBMBT also satisfies an universal principle: it appears naturally in the large limit of a discrete model. In a second part of my talk, I will quickly explain how one can develop a stochastic calculus for FBMBT. This is a difficult question since the trajectories of FBMBT are more irregular than Brownian motion. The talk will be accessible to a non-specialist audience.