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Transboundary Pollution and abatement

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Chapter 1

TRANSBOUNDARY POLLUTION AND ABATEMENT

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Abstract

Adopting clean technologies is a long term process which requires structural changes in production and consumption habits. In the present paper, we focus more on short term issues related to pollution reduction and analyze a pollution abatement game in a 2-country dynamic model. Transboundary pollution is treated as a common state variable while pollution reduction is reached via abatement rather than the adoption of cleaner technologies. Symmetric open-loop and Markovian Nash equilibrium are studied and compared as well as the analysis of Markovian strategies of more than two countries case around the steady state.

1 Introduction

On average, every week a new coal-fired power plant opens somewhere in China, big enough to supply energy to a one-million inhabitants city [9]. To a lesser extend, the same is true in India where coal demand for electricity is rising strongly. More generally, the appetite for energy in general, and for coal demand in particular, shows no sign of curbing. This has lead the International Energy Agency to forecast that coal would retain about 25

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per cent of its share of world energy mix, a gloomy prospect for dimming greenhouse gas emissions as coal is by far the most carbon dioxide (CO2) intensive energy source.

Recently, however the expression "clean coal technology" has appeared repeatedly in the media, referring to technologies being developed that aim to reduce the environmental impact of coal energy generation. Scientists refer to "carbon capture and storage" (CCS) to describe the process of mitigating the contribution of fossil fuel emissions to climate change, based on catching CO2 from large point sources. CCS does not prevent the formation of CO2 but reduces its emissions in the atmosphere. In this regard, it is often considered as a desirable complement to CO2 mitigation policies, as evidenced by the 2005 special report of the Intergovernmental Panel on Climate Change (IPCC) on these matters.

Nevertheless, due to the absence of supranational institution able to enforce international legislation, the environmental interdependence of greenhouse gas (GHG) reduction has slowed down the implementation of large scale transboundary policy such as carbon capture. Concern about such issues has spurred a strand of literature analyzing environmental issues in a strategic framework (see [6]). In this context, differential games are prone to tackle issues such as negotiations outcomes on bilateral pollution control agreements and the incentives to deviate therefrom.

In the present study, we will focus our attention on a dynamic model of international pollution reduction, explicitly developing the strategic behavior countries can adopt. In particular, special emphasis will be devoted to (i) two alternative non-cooperative scenarios, and (ii) the opportunity of the number of participants in international pollution reduction agreements.

Our paper departs from the existing literature in that it explicitly refers to capture and storage of GHG emissions rather than pollution reduction at the source, as is often the case in existing models. This has important consequences on our modeling strategy: first, pollution is predetermined in the model, and the decision is rather on the amount of cleaning up of this pollution. A corollary of this is that pollution reduction does not imply a direct reduction of output in the production process.

Second, we do not have to introduce some delay mechanism taking account of the adoption period for newer technologies. This is especially relevant when considering less developed countries.

Third, nature's regeneration capacity is explicitly introduced in the pollution equation. The self-regeneration capacity of nature is often neglected in the literature[1]. It is, nevertheless, of tremendous importance in the present context as the regeneration capacity acts similarly as pollution drops on mitigating environmental degradation.

Fourth, we propose two alternative equilibrium concepts– the open-loop strategy and the Markovian (perfect) Nash equilibrium (as in [6]). In the former approach, countries commit to an abatement level over time, independently of the state of the world. In the latter approach, however, abatement strategies may vary in time, according to the pollution

[1]From a theoretical perspective, most studies coping with the issue of the relationship between economic development and environmental quality are constructed as if pollution were just another input in the production function and ignore the possibility that different countries may be endowed with varying capacities to absorb pollution; see, for instance, Stokey's [12] fundamental contribution to the theoretical modeling. A notable exception in this regard is Aghion and Howitt ([1], Chap. 5),who introduce a potential rate of regeneration in their environmental quality equation; see also [2]

level.

Fifth, we extend our model to the case where more than two countries are involved. Increasing the number of participants in the game implicitly hints at the (dis)advantages of multilateralism in negotiations related to the environment.

Last, our contribution is closely related to the seminal work of Dockner et al. (see [5] and [6]). However, these later authors pay more attention to the existence, properties, and multiplicity of stationary steady states via Hamilton-Jacob-Bellman (HJB thereafter) equations whereas stationary equilibrium analysis can not be at the center of our preoccupations as issues linked to carbon capture and storage are relevant for short and medium term, and thus the dynamics are fundamental. Therefore, we present the transitionary dynamics via Pontryagin's maximum principle. Nevertheless, and for ease of comparison with existing literature, HJB results are also displayed. Finally, we assume that the players could adopt open-loop strategies, i.e. no information acquired during the game is used and thus there is no response to changes in the current stock [5].

1.1 The economics of carbon capture and sequestration

Worldwide GHG emissions come primarily from the combustion of fossil fuels, of which power stations and transport are the largest contributors. According to projections of the US Energy Information Administration, worldwide installed electricity generating capacity will almost double until 2030, and coal-fired plants will account for about a third of total electricity capacity ([9], [11]). This results essentially from its lower production costs, but also because most renewable energy sources lack reliability, e.g., wind or solar energy can only supply electricity if there is wind or sun! Thus, reducing emissions from these coal or oil fired power plants will only be feasible through carbon capture.

While mitigating emissions from transport can essentially be achieved either through the reduction of road transport or the increase in its efficiency, power stations' emission toll can additionally be improved through capturing and storing their gas, i.e., so called carbon capture and storage (CCS).

CCS has attracted considerable attention since the beginning of the 1990s as a sensible possibility of mitigating carbon dioxide's effect on global climate change. While carbon capture in the transport sector would be unfeasible due to the multiplicity of emission sources, power plants[2] being large point sources, make it suitable to capture carbon dioxide.

CCS is probably among the most realistic means of reducing emissions in the closer future as the world is likely to continue to use large quantities of fossil fuels for decades, despite the increasing share of renewable and nuclear energy in electricity generation. Renewable energy is more likely to develop for new supplementary energy generation.

There are several processes available for the capture of carbon dioxide. Essentially, the technology, applied usually to coal-fired power plants to capture the gas, compress it and bury it in underground storage sites or in the ocean. For the transport of carbon dioxide to the storage site, either pipelines can be used for large amounts, or alternatively ships for

[2]Actually, besides power plants, other industrial activities (such as cement industry, iron and steel industry, petrochemical industries and refineries) constitute stationary carbon dioxide sources and qualify thus for CCS. Nevertheless, fossil fuel based power generation accounts for about three quarters of total localized carbon dioxide emissions. Thus, we will essentially refer to power plants henceforth.

large distances overseas. This, however, means that plants equipped with a CCS system will need up to 40 per cent more energy than an equivalent plant without CCS, which on balance amounts to a net carbon dioxide emissions reduction of approximately 80-90 per cent [10].

In Japan, a grouping of firms is currently working with the government on the feasibility of CCS projects, notably in the seabed. In Europe, carbon capture and geological storage is part of the 2007 energy package which sets out the general strategy as well as the regulatory framework with the objective of lifting unwarranted barriers to CCS in existing legislation [8].

The paper is organized as follows: Section 2 presents the model while sections 3 and 4 explain the theoretical results for different strategies and for differing strategies. Section 5 concludes.

2 The model

Suppose there are two countries, country i and j. Both produce some consumption goods with pollution as a byproduct. These two cross boundary countries share the same pollution state, $x(t)$, which is given by the following equation

$$\dot{x}(t) = E(t) - (u_i + u_j)\sqrt{x(t)} - \delta x(t), \;\; t \geq 0 \tag{1}$$

with initial condition $x(0)$ a given positive constant, and parameter $\delta \in [0, 1]$ measuring the pollution absorption rate of nature. $E(t) = E_i(t) + E_j(t)$ is a known positive function of pollution emission. In the long run, $E(t)$ may be constant, increasing, or decreasing at a constant rate. u_i and u_j are abatement rates of country i and j, respectively. Notice that if abatement is zero, pollution will increase much faster. Furthermore, if emissions are over the absorption rate of nature itself, pollution will increase without bound. Expression $-(u_i + u_j)\sqrt{x(t)}$ shows that the higher pollution state is, the more efficient is one unit of abatement. We take the square root terms only for analytical ease.[3]

Facing the pollution-abatement problem, the two countries need to choose their abatement rate u_l, $l = i, j$, to maximize their utility

$$\max_{u_l} \int_0^{\overline{T}} e^{-r_l t} \left(-x(t) - \frac{\alpha_l}{2} u_l^2 \right) dt + S(x(\overline{T})), \;\; l = i, j, \tag{2}$$

subject to the state constraint (1) where $r_l \in [0, 1)$ is the time preference, and positive constant α_l is the adjustment cost coefficient. We would consider $\overline{T} \leq \infty$, where $S(x(\infty)) = 0$ and in finite time $\overline{T} = T$, $S(x(T))$ is a given known positive function with $S_x < 0$.

This differs from [6], where the emissions are choice variables of the two countries. In our framework, they are exogenously given while the abatements are endogenously chosen.

3 Solution

In this section, we will study two different strategies under different frameworks. In a symmetric framework, we first study the open-loop strategy where at the beginning of the game,

[3] See page 53 of [4] for further discussions.

both countries commit to abatement levels $u_l = u_l(t)$ for each period in time which is independent of the state of the world. Then, we study the Markovian perfect Nash equilibrium where the abatement strategies change basing on the state of pollution, $u_l = u_l(x(t), t)$. Both Pontryagin maximum principle and Hamilton-Jacob-Bellman equation will be presented and with a comparison of the results from these two different methods.

3.1 Open-loop strategy

Suppose the two countries are identical and both countries commit their abatement strategy only basing on time t rather than the pollution state. Then player i's problem is looking for $u_i(t)$ to maximize her utility,

$$\max_{u_i(t)} \int_0^{\overline{T}} e^{-r_i t} \left(-x(t) - \frac{\alpha_i}{2} u_i^2 \right) dt, \tag{3}$$

subject to the state constraint (1).

It is easy to see that player i's Hamiltonian function[4] is

$$H_i(x, u_i, \lambda_i, t) = \left(-x(t) - \frac{\alpha_i}{2} u_i^2 \right) + \lambda_i \left[E(t) - (u_i + u_j^*(t))\sqrt{x(t)} - \delta x(t) \right]$$

where y^* represents the optimal strategy of variable y.

Pontryagin's maximum principle shows that

$$u_i(t) = -\frac{\lambda_i(t)\sqrt{x(t)}}{\alpha_i}, \tag{4}$$

and the costate variable, which serves as the shadow value, follows

$$\dot{\lambda}_i(t) = r_i \lambda_i(t) - \frac{\partial H_i}{\partial x} = (r_i + \delta)\lambda_i + \frac{u_i + u_j^*}{2\sqrt{x}} \lambda_i + 1 \tag{5}$$

with transversality condition

$$\lim_{t \to \infty} e^{-r_i t} \lambda_i(t) x(t) = 0, \tag{6}$$

or

$$\lambda(\overline{T}) = S_x(x^*(\overline{T})). \tag{7}$$

Remark. From (5), we can see that the natural absorption rate δ has the same effect as the time preference r_i. In the following calculation, we will set $\delta = 0$, while in the interpretation, we may pick up the effect of this absorption rate.[5]

Furthermore, by the symmetric assumption, $r_i = r_j = r,\ \alpha_i = \alpha_j = \alpha$, we have

$$u_i(t) = u_j(t) = u(t),\ \lambda_i(t) = \lambda_j(t) = \lambda(t),$$

[4] Notice there is an exogenous function $E(t)$ which may depend on time t, especially during the short run. The HJB equation will show up as a real partial differential equation with a term V_t, given there is no boundary (or transversality) condition. In general, it is more difficult to find solutions, and it is no longer stationary as most of the cases. From this point of view, with $E(t)$ a given function of time t, Pontryagin's maximum principle provides richer results.

[5] Set r_i as $r_i + \delta$ in the following of this paper.

so do the optimal values. Therefore, the above analysis yields the following system

$$\begin{cases} u^*(t) = -\frac{\lambda(t)\sqrt{x(t)}}{\alpha}, \\ \dot{x} = E(t) + \frac{2\lambda x(t)}{\alpha}, \\ \dot{\lambda} = 1 + r\lambda - \frac{\lambda^2}{\alpha}, \end{cases} \tag{8}$$

with initial condition and transversality condition (6). The shadow price λ (which is a Riccati equation) can be solved explicitly while pollution state x is a linear equation. Hence, in the appendix, we obtain the explicit solution for the open loop problem

$$\begin{cases} \lambda(t) & = \overline{\lambda} = \dfrac{\alpha r - \sqrt{(\alpha r)^2 + 4\alpha}}{2}, \quad \overline{T} = \infty, \\ x_o(t) & = e^{\frac{2\overline{\lambda}}{\alpha}t}\left[x(0) + \displaystyle\int_0^t e^{-\frac{2\overline{\lambda}}{\alpha}s}E(s)ds\right], \quad \overline{T} = \infty, \\ \lambda(t) & = A + \frac{r\alpha}{2} - \frac{2A}{C_1 e^{\frac{2At}{\alpha}} + 1}, \quad \overline{T} = T < \infty, \\ x_o(t) & = e^{\int_0^t \frac{2\lambda(s)}{\alpha}ds}\left[x(0) + \displaystyle\int_0^T e^{-\frac{2\lambda(s)}{\alpha}ds}E(\tau)d\tau\right], \quad \overline{T} = T < \infty, \end{cases} \tag{9}$$

with $C_1 = \dfrac{2A - r\alpha + 2S_x(x(T))}{2A + r\alpha - 2S_x(x(T))}e^{-\frac{2A}{\alpha}T}$ and $A = \sqrt{\alpha + (\frac{r\alpha}{2})^2}$.

The above explicit solutions show the short run dynamics of the system under open-loop strategy with infinite time horizon, where the shadow price is always constant over time. Whereas, if there is a finite time target for the pollution level, $S(x(T))$, the shadow value, $\lambda(t)$, changes over time to match this final target.

Remark. In the following, we will only study the case where $\overline{T} = \infty$ since the explicit solution of other strategies can be obtained by the same arguments as above.

Straightforwardly, from the explicit solution (9), it follows

$$\frac{\partial\lambda}{\partial r} = \frac{\alpha}{2}\left[1 - \frac{\alpha r}{\sqrt{(\alpha r)^2 + 4\alpha}}\right] > 0,$$

which reads that people care less about pollution state if r is larger or if people are more impatient (or people care less about the pollution state if nature's absorption rate δ is larger) by recalling that λ is negative.

Substituting (4) into the Hamiltonian function, we have

$$H^*(x, \lambda, t) = \max_u H(x, u, \lambda, t) = -\left[1 - \frac{\lambda^2}{2}\right]x + \lambda\left[E(t) + \left(\frac{2\lambda}{\alpha} - \delta\right)x\right]$$

which is linear in state variable x and, hence, concave with respect to x. Therefore, the solution from Pontryagin's maximum principle is not only necessary but also sufficient (see, for example, Theorem 3.2 in [4]).

Now, we study the long run balanced growth path where all the endogenous variables grow at constant rates. These constant rates could be positive, or negative, or zero. If all the constant rates are zero, we call it "a steady state".

From the dynamic equation $\frac{\dot{\lambda}}{\lambda} = \frac{1}{\lambda} + r - \frac{\lambda}{\alpha}$ and its short run solution, we have $\lambda = \overline{\lambda} = \lambda^*$ for any time $t > 0$. Recall that λ measures the shadow value of pollution; the value of people measuring pollution is a constant and no longer changing over time, though the state of pollution x and the emission E may still change over time. Indeed, $\frac{\dot{x}}{x} = \frac{E(t)}{x(t)} + \frac{2\lambda^*}{\alpha}$ shows that $\frac{\dot{x}}{x}$ is constant if, and only if, the emissions and pollution grow at the same rate given that λ^* is a constant. This can happen if $u^* = -\frac{\lambda^*\sqrt{x^*}}{\alpha}$. We conclude the above analysis by the following proposition:

Proposition 1 *In the long run,*

$$\lambda^* = \overline{\lambda} = \frac{\alpha r - \sqrt{(\alpha r)^2 + 4\alpha}}{2} < 0.$$

Moreover, there are three possibilities based on the emission of pollution.

(1) If pollution emission $E = \overline{E}$ is a constant, there is a unique steady state, where states of pollution and abatement are constants and give by

$$x_o^* = -\frac{\alpha \overline{E}}{2\lambda^*}, \quad u_o^* = -\frac{\lambda^*\sqrt{x^*}}{\alpha}.$$

(2) If pollution emissions $E = E(t)$ are increasing (or decreasing) over time with constant growth rate g_E (which could be negative or positive), the pollution state will increase (or decrease) at the same rate g_E while the abatement increase (or decrease) at rate $\frac{g_E}{2}$. But the shadow value of the pollution is the same constant as above.

(3) If pollution emission $E = E(t)$ is neither of the above cases, then there is no balanced growth path nor a steady state.

For the first case in the above proposition, we can also study the convergence to the steady state. Suppose $E = \overline{E}$, combining Proposition 1 and dynamic system (8), through the standard Jacobian matrix analysis,[6] we have that there are two eigenvalues of the dynamics system (8) in which one is positive and one is negative. This shows that the steady state is a saddle point. As a conclusion, we have the following results.

Proposition 2 *If in the long run, the pollution emission $E = \overline{E}$ is a constant, then there exists a convergence path, which locally converges to the saddle point steady state mentioned in Proposition 1. If the pollution emission $E = \overline{E}$ is always a constant, then the saddle path convergence is global. The convergence speed is $v_o = \frac{2|\lambda^*|}{\alpha}$.*

[6]The Jacobian matrix is

$$J^o = \begin{pmatrix} \frac{2\lambda^*}{\alpha} & \frac{2x_o^*}{\alpha} \\ 0 & -\frac{2\lambda^*}{\alpha} + r \end{pmatrix}.$$

Denote the two eigenvalues of this Jacobian is $\xi_{1,2}$. It is easy to check

$$\xi_1 = \frac{2\lambda^*}{\alpha} < 0, \quad \xi_2 = r - \frac{2\lambda^*}{\alpha} > 0,$$

due to $\lambda^* < 0$.

3.2 Markovian (Perfect) Nash Equilibrium

Now suppose that both countries can change the abatement strategies based on the pollution state. Then each player is facing the same problem as above by choosing the abatement strategy $u_i = u_i(x(t), t)$ for a given optimal strategy of player j, $u_j^*(x(t), t)$. In the sequel, we will employ two different methods to look at this Markovian Nash Equilibrium and Markovian perfect Nash Equilibrium.

3.2.1 Pontryagin's Maximum principle - explicit solution

Player i's Hamiltonian function is

$$H_i(x, u_i, u_j^*(x,t), \phi_i, t) = \left(-x(t) - \frac{\alpha_i}{2}u_i^2\right) + \phi_i\left[E(t) - (u_i + u_j^*(x(t),t))\sqrt{x(t)}\right]$$

where ϕ_i is player i's costate variable.

Pontryagin's maximum principle shows that

$$u_i(t) = -\frac{\phi_i(t)\sqrt{x(t)}}{\alpha_i}, \tag{10}$$

and

$$\dot{\phi}_i(t) = r_i\phi_i(t) - \frac{\partial H_i}{\partial x} = r_i\phi_i + 1 + \frac{u_i + u_j^*}{2\sqrt{x}}\phi_i + \phi_i\sqrt{x}\frac{\partial u_j^*}{\partial x} \tag{11}$$

with transversality condition

$$\lim_{t\to\infty} e^{-r_i t}\phi_i(t)x(t) = 0. \tag{12}$$

Given symmetry assumption, it follows

$$\begin{cases} u_i^* = u_j^* = u^* = -\frac{\phi(t)\sqrt{x}}{\alpha}, \\ \dot{x} = E(t) + \frac{2\phi x}{\alpha}, \\ \dot{\phi} = 1 + r\phi - \frac{3\phi^2}{2\alpha} \end{cases} \tag{13}$$

with initial condition and transversality condition (12).

Using similar methods to obtaining (9), the explicit solution to the Markovian strategy is given by

$$\phi(t) = \overline{\phi} = \frac{1}{3}\left(\alpha r - \sqrt{(\alpha r)^2 + 6\alpha}\right) (< 0), \quad x_m(t) = e^{\frac{2\overline{\phi}}{\alpha}t}\left[x(0) + \int_0^t e^{-\frac{2\overline{\phi}}{\alpha}s}E(s)ds\right]. \tag{14}$$

In the long run, similarly to the open loop case, we obtain the Markovian Nash equilibrium which we present in the following proposition.

Proposition 3 *(1) If in the long run pollution emissions $E = \overline{E}$ are constant, there is a unique steady state where the state of pollution, the shadow value of pollution, and abatement are constants and given by*

$$\phi^* = \frac{r\alpha - \sqrt{(r\alpha)^2 + 6\alpha}}{3} < 0, \ \ x_m^* = -\frac{\alpha\overline{E}}{2\phi^*}, \ \ u_m^* = -\frac{\phi^*\sqrt{x_m^*}}{\alpha}.$$

Furthermore, there is a local saddle path along which the dynamics converge to this steady state. Convergence speed will be $v_m = \frac{2|\phi^*|}{\alpha}$.

(2) and (3) as in Proposition 1 similarly hold.

The difference between the open loop strategy and the above Markovian Nash strategy lies in the fact that in the open loop framework both countries commit at the beginning of the game and do not change after the game starts while, in the Markovian case, both countries can adjust their strategies based on the real pollution state. In the appendix, we prove the following results:

Proposition 4 *Suppose pollution emissions are constants (at least in the long run), then*

$$\lambda^* < \phi^* < 0, \quad x_o^* < x_m^*, \quad u_m^* < u_o^*$$

which means that the pollution level is higher and abatement is lower in the Markovian Nash equilibrium than in the open loop equilibrium. That is, if the two players can change the strategies basing on the state of the world, there is free riding compared to the case where there are changing strategies across time only.

From the above comparison results, it is easy to see that the convergence speed checks

$$v_m < v_o$$

which reads that in the open loop case, people care more about the pollution. Also the converges to its steady state level is faster than in the closed loop, where slow convergence happens with a higher pollution level at the steady state.

3.2.2 Hamilton-Jacob-Bellman Equation - infinite number of solutions

In this section, we use the Hamilton-Jacob-Bellman equation to look for further other feedback strategies. Denote the player i's Bellman value function as $V_i(x)$ which checks the following HJB equation

$$r_i V_i(x) - (V_i)_t = \max_{u_i} \left\{ \left(-x(t) - \frac{\alpha_i}{2} u_i^2 \right) + (V_i)_x \left[E(t) - (u_i + u_j(x))\sqrt{x} - \delta x \right] \right\}, \tag{15}$$

where $(V_i)_t$ and $(V_i)_x$ are partial derivatives in terms of t and x, respectively.

In the following, we only consider a special case, where $\overline{T} = \infty,\ E(t) = \overline{E}$ is a constant. Then, we can take stationary values only, i.e. $(V_i)_t = 0$. With this assumption, the above equilibrium is Markovian perfect Nash Equilibrium.

The first order condition with respect to u_i on the right hand side of (15) reads

$$u_i = -\frac{(V_i)_x \sqrt{x}}{\alpha}.$$

By symmetry, it follows

$$V_B(x) = V_i(x) = V_j(x) = -\frac{\alpha u_B}{\sqrt{x}}, \quad u_B = u_i = u_j.$$

Then the HJB equation can be rewritten as

$$rV(x) = -x - \frac{\alpha}{2}u^2 - \frac{\alpha \overline{E} u(x)}{\sqrt{x}} + 2\alpha u^2 + \alpha\delta u\sqrt{x}. \tag{16}$$

Following exactly the same arguments as in [5] and [6], we can prove that there are infinite solutions to the above HJB equation and, hence, there are infinite feedback strategies. Moreover, there is an interval of pollution levels characterized by an asymptotic stable steady state. The above feedback strategy from Pontryagin's maximum principle is one of the special cases of the infinite feedback strategies where $V(x) = \overline{\phi}x + \overline{\phi E}$ and $u(x) = -\frac{\phi\sqrt{x}}{\alpha}$.

4 More than two players

In this section, we study a case where there are N players, $N \geq 2$. With the same calculation as in the previous section, we show the results of the open-loop strategy and feedback strategy via Pontryagin's maximum principle.

Open-loop strategy

Keeping the same notation as in the previous section except we denote the $N-$player case by superscript N, then the open-loop optimal strategy is

$$u_o^N = -\frac{\lambda^N \sqrt{x_o^N}}{\alpha}$$

and the dynamics of state and costate equations are

$$\begin{cases} \dot{x_o^N} = E(t) + \frac{N}{\alpha}\lambda^N x_o^N, \\ \dot{\lambda^N} = 1 + r\lambda^N - \frac{N}{2\alpha}\left(\lambda^N\right)^2 \end{cases} \tag{17}$$

with initial condition and transversality condition.

Following the same calculations as in the previous section, we get the following explicit solution to the above dynamics system

$$\begin{cases} \overline{\lambda^N} = \lambda^N(t) = \dfrac{\alpha r}{N} - \sqrt{\dfrac{2\alpha}{N} + \left(\dfrac{\alpha r}{N}\right)^2}, \\ x_o^N(t) = e^{\frac{N\overline{\lambda^N}}{\alpha}t}\left[x(0) + \displaystyle\int_0^t e^{-\frac{N\overline{\lambda^N}}{\alpha}s}E(s)ds\right]. \end{cases} \tag{18}$$

In the case where there is constant emission of pollution $E(t) = \overline{E} = N\widehat{E}$ and where $\widehat{E}$ is per player's emission of pollution, the steady state level of pollution is

$$\overline{x_o^N} = -\frac{\alpha\overline{E}}{N\overline{\lambda^N}}. \tag{19}$$

4.1 Effect of N

With the above explicit solution, we can study the effect of the number of players on the pollution level and optimal strategy. Straightforwardly, we have

$$\frac{\partial\overline{\lambda^N}}{\partial N}=\frac{(\alpha N+(\alpha r)^2)-\alpha r\sqrt{2\alpha N+(\alpha r)^2}}{N^2\sqrt{2\alpha N+(\alpha r)^2}}>0 \tag{20}$$

which reads that with the increase in the number of players, each player cares less about environmental pollution.

Moreover,

$$\frac{\partial\overline{x_o^N}}{\partial N}=\frac{\alpha\overline{E^i}}{\overline{\lambda^N}^2}\frac{\partial\overline{\lambda^N}}{\partial N}=-\frac{\overline{x_o^N}}{\overline{\lambda^N}}\frac{\partial\overline{\lambda^N}}{\partial N}>0$$

which follows from (20). Since every player cares less about pollution, the pollution level is increasing with the number of players.

Combining the above two expressions together, it follows

$$\frac{\partial u_o^N}{\partial N}=-\frac{1}{\alpha}\left[\sqrt{\overline{x_o^N}}\frac{\partial\overline{\lambda^N}}{\partial N}+\frac{\overline{\lambda^N}}{2\sqrt{\overline{x_o^N}}}\frac{\partial\overline{x_o^N}}{\partial N}\right]=-\frac{\sqrt{\overline{x_o^N}}}{2\alpha}\frac{\partial\overline{\lambda^N}}{\partial N}<0$$

which states that with the increasing number of players, everyone would like to free ride on the other's efforts to clear up the pollution. Therefore, the abatement is decreasing in terms of the number of players.

The result demonstrates that dealing with the pollution problem above, small groups, or communities, or states, rather than big groups, do better by committing their strategies.

4.2 Feedback Strategy

Similarly, the optimal feedback strategy is

$$u_p^N(x)=-\frac{\phi^N\sqrt{x_p^N}}{\alpha}$$

and the dynamics are

$$\begin{cases}\dot{x_p^N}=E(t)+\frac{N}{\alpha}\phi^N x_p^N,\\ \dot{\phi^N}=1+r\phi^N-\frac{2N-1}{2\alpha}\left(\phi^N\right)^2\end{cases} \tag{21}$$

with initial condition and transversality condition. Hence, the explicit solution of the above dynamic system is

$$\begin{cases}\overline{\phi^N}=\phi^N(t)=\dfrac{\alpha r}{2N-1}-\sqrt{\dfrac{2\alpha}{2N-1}+\left(\dfrac{\alpha r}{2N-1}\right)^2},\\ x_p^N(t)=e^{\frac{N\overline{\phi^N}}{\alpha}t}\left[x(0)+\displaystyle\int_0^t e^{-\frac{N\overline{\phi^N}}{\alpha}s}E(s)ds\right].\end{cases} \tag{22}$$

Constant emission of pollution $E(t) = \overline{E}$ leads to the steady state level of pollution

$$\overline{x_p^N} = -\frac{\alpha \overline{E}}{N\overline{\phi^N}}. \tag{23}$$

Also similar results as in the open-loop case can be proven. As a by product of (20), it yields

$$\overline{\lambda^N} < \overline{\phi^N},$$

given $N < 2N - 1$ for $N \geq 2$. Hence, the feedback information gives a worse scenario in this across border pollution problem: the more players are playing, the worse environmental pollution outcome will be.

5 Conclusion

To our knowledge, this is the first contribution tackling the carbon capture and storage (CCS) problem in a multi-country setting. Previous studies, either in a single country framework or in a strategic framework as ours, have essentially focused on pollution mitigation, which should be considered as a long term strategies compared to CCS.

Our results show that feedback strategies lead to free riding compared to the open loop strategy. Furthermore, under a non-cooperative setting, fewer players lead to a more desirable result than in the case where there are more players. This results from the higher incentives to free ride when the number of players increases.

Further research perspectives might introduce nature's regeneration capacity in an endogenous way which would then allow us to cope not only with carbon capture and storage but also carbon sequestration through crop selection.

References

[1] Aghion, P. and Howitt, P., 1998, Endogenous Growth Theory. Cambridge, MA. MIT Press.

[2] Bertinelli, L, Strobl E., and Zou B., 2008, Economic development and environmental quality: A reassessment in light of nature's self-regeneration capacity, *Econological Economics* 66, 371-378.

[3] Clemhout S. and H. Wan Jr. 1985, Dynamic common property resources and environmental problems, *Journal of Optimization Theory and Application* 46, 471-481.

[4] Dockner E., Jorgensen S., Van Long N., and Sorger G., 2000, *Differential games in economics and management*, Cambridge University Press.

[5] Dockner E. and Sorger G., 1996, Existence and properties of equilibria for a dynamic game on productive assets, *Journal of Economic Theory* 71, 209-227.

[6] Dockner E. and Van Long N., 1993, International pollution control: Cooperative versus noncooperative strategies, *J. Of Environmental Economics and Management*, 24, 13-29.

[7] *Energy Information Administration (EIA)*, 2006, International Energy Outlook 2006, US Dept. of Energy, Washingtion, D.C.

[8] *European Commission (EC)*, 2007, Sustainable power generation from fossil fuels: aiming for near-zero emissions from coal after 2020, Communication from the Commission to the Council and the European parliament,

$http://ec.europa.eu/environment/climat/ccs/pdf/com2006_0843_en.pdf$.

[9] *International Energy Agency (IEA)*, 2008, World Energy Outlook 2008, IEA publications. "

[10] *Intergovernmental panel on climate change (IPCC)*, 2007, Carbon dioxide capture and storage, Special report, Cambridge University Press. "

[11] Liang X., J. Li, J. Gibbons and D. Reiner, 2007, Financing capture ready coal-fired power plants in China by issuing capture options, bEPRG 0728 and CWPE 0761.

[12] Stokey N.L., 1998, Are there limits to growth?, *International Economic Review*, 39(1), 1–31.

Appendix

5.1 Obtaining solution (9)

Rewriting the costate equation as

$$\frac{d\lambda}{dt} = 1 + r\lambda - \frac{\lambda^2}{\alpha} = \frac{1}{\alpha}\left(\alpha + \alpha r\lambda - \lambda^2\right) = \frac{1}{\alpha}\left[A^2 - \left(\lambda - \frac{\alpha r}{2}\right)^2\right]$$

where $A^2 = \alpha + \left(\frac{\alpha r}{2}\right)^2$. Then, the costate equation is

$$\frac{d\lambda}{A^2 - (\lambda - \frac{\alpha r}{2})^2} = \frac{dt}{\alpha}.$$

Denoting $y = \lambda - \frac{\alpha r}{2}$, we can rewrite the above equation as the following:

$$\frac{1}{2A}\left(\frac{dy}{A-y} + \frac{dy}{A+y}\right) = \frac{dt}{\alpha}$$

with $A = \sqrt{\alpha + (\frac{\alpha r}{2})^2}$.

Straightforward, the above equation leads to the following solution

$$\ln\left(\frac{A+y}{A-y}\right) = \frac{2A}{\alpha}t + C$$

where C is a constant and undetermined.

Rearranging the terms, we find

$$y(t) = \frac{\left(C_1 e^{\frac{2At}{\alpha}} - 1\right) A}{\left(C_1 e^{\frac{2At}{\alpha}} + 1\right)} \text{ or } \lambda(t) = A + \frac{\alpha r}{2} - \frac{2A}{C_1 e^{\frac{2At}{\alpha}} + 1}$$

where C_1 is an undetermined constant.

Notice that for $\overline{T} = \infty$, at any time $t \geq 0$, we must have $\lambda(t) \leq 0$, even for $t \to \infty$. However, in the above explicit solution, we notice, if $C_1 \neq 0$, then

$$\lim_{t\to\infty} \lambda(t) = A + \frac{\alpha r}{2} - \lim_{t\to\infty} \frac{2A}{C_1 e^{\frac{2At}{\alpha}} + 1} = A + \frac{\alpha r}{2} > 0$$

which is impossible. Therefore, $C_1 = 0$. Hence the explicit solution to the costate equation is

$$\lambda(t) = \frac{\alpha r}{2} - A$$

which is always a constant and negative, and we denote this constant by $\overline{\lambda}$.

For $\overline{T} = T < \infty$, the transversality condition is $\lambda(T) = S_x(x^*(T))$ which gives

$$C_1 = \frac{2A - r\alpha + 2S_x(x(T))}{2A + r\alpha - 2S_x(x(T))} e^{-\frac{2A}{\alpha}T},$$

and promises that $\lambda(t) \leq 0, \forall t \in [0, T]$.

As to the state equation, it is easier to multiply the factor $e^{-\frac{2\overline{\lambda}}{\alpha}t}$ on both sides. Straightforward, it yields, for $\overline{T} = \infty$,

$$x_o(t) = e^{\frac{2\overline{\lambda}}{\alpha}t} \left[x(0) + \int_0^t e^{-\frac{2\overline{\lambda}}{\alpha}s} E(s) ds \right],$$

or for $\overline{T} = T < \infty$,

$$x_o(t) = e^{\int_0^t \frac{2\lambda(s)}{\alpha} ds} \left[x(0) + \int_0^\tau e^{-\frac{2\lambda(s)}{\alpha} ds} E(\tau) d\tau \right].$$

That finishes the proof of solution (9).

5.2 Proof of Proposition 4

Comparing the difference between the two absolute value of λ^* and ϕ^*, it follows

$$\begin{aligned} \mid \lambda^* \mid - \mid \phi^* \mid &= \frac{\sqrt{(r\alpha)^2 + 4\alpha} - r\alpha}{2} - \frac{\sqrt{(r\alpha)^2 + 6\alpha} - r\alpha}{3} \\ &= \frac{1}{6} \left[\sqrt{(3r\alpha)^2 + 36\alpha} - \sqrt{(2r\alpha)^2 + 24\alpha} - r\alpha \right], \end{aligned}$$

So, $\mid \lambda^* \mid > \mid \phi^* \mid$ if and only if

$$\sqrt{(3r\alpha)^2 + 36\alpha} > \sqrt{(2r\alpha)^2 + 24\alpha} + r\alpha.$$

Squaring on both sides and rearranging terms, it follows that $\mid \lambda^* \mid > \mid \phi^* \mid$ if and only if

$$(12\alpha)^2 > 0$$

which is always true.

We finish the proof.

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